A Comparative Study to Investigate the Effect of the Number of Sub-Lots in a Multi-Stage Multi-Product Flowshop Lot-Streaming Problem with Consistent Sub-lot sizes: A Case Study

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Abstract
Lot-streaming in flowshop models is intensely studied and numerous results are obtained and reported in the literature. The maximum number of sub-lots is relaxed in many of these published research works. However, in real-life situations, limitation on the number and capacity of material handling facilities or technical considerations may constrain decision makers to plan for moving too many of sub-lots between machines on the shop floor. To tackle this problem, some researchers have put an upper bound on the number of sub-lots and/or on their size. In this paper, we aim at investigating the impact of a maximum number of sub-lots on makespan. To this goal, a comparative study is conducted to find out if with the assumption of consistent-sized sub-lots, equal and unequal sub-lot sizes affect makespan. Then using discrete-event simulation and a generic MIP model, a relatively good policy is obtained and applied to a case study to minimize the makespan.

Keywords:  
Scheduling, Lot-Streaming, Flowshop, Mixed integer linear programming.

1. Introduction
Lot-streaming is a well-established problem in the flowshop scheduling literature. Lot-streaming means dividing a lot into two or more sub-lots to be processed through the production line simultaneously in order to improve the performance of the system [1]. Lot-streaming has been applied to reduce makespan, average cycle time and average work-in-process [2]. Lot-streaming is also called batch splitting in the literature. Since these problems are NP Hard for large sizes, many researches have been conducted using an appropriate heuristic method to find the optimal or near optimal sub-lot sizes and sub-lot sequencing. Geometric sub-lot sizes presented by Potts and Baker [3] are proved to be optimal for a single-product two-stage flowshop with continuous sub-lot sizes. This idea then was developed and investigated by many researchers such as Alfieri et al. [4] for lot-attached setup times, which means that the setup is performed on a machine just before performing production process on that machine. The flowshop lot-streaming problem is more complicated for multi-stage multi-product production lines than it is for single-stage or single-product production lines. The travelling salesman problem is proved to be optimal by Kumar et al. [5] for lot-streaming flowshop with detached setup times and continuous-sized sub-lots. A genetic algorithm (GA) is applied to solve the travelling salesman problem.

Mathematical modeling is the other approach applied in the literature. A mixed integer linear programming model for scheduling and lot-streaming with sequence-dependent setup times is presented in [6]. The goal is to minimize the makespan, and a GA is applied to find a near to optimal solution in a reasonable time. Sarin and Jaiprakash reviewed mathematical models under different assumptions for lot-streaming flowshop problems in the second chapter of their book [7].
Many of the conducted researches have concentrated on heuristic and meta-heuristic approaches to solve the problem under several conditions. Furthermore, some helpful policies and methods are developed for different flowshop configurations. However, to the best of our knowledge, no research has been conducted to investigate the effect of number of sub-lots and sub-lot sizes where the number of sub-lots is known in advance and the sub-lot sizes are consistent and continuous.

In this article, we aim at investigating lot-streaming problem for a multi-product multi-stage production line to find a relatively good policy for a company located in Southwestern Ontario, Canada. The objective function is to minimize the makespan, which is the case for make-to-order production systems where a product due date is of importance. This examination is performed using an MIP model.

Discrete-event simulation is a technique widely in use to tackle uncertainty in complex systems where analytical models are difficult to build or to solve. Using a simulation model built for a shop floor, any operational plan can be easily investigated and the impact of implementing this plan would be reflected in the performance measures of the system. In the case study presented in this article, the proposed lot-streaming and sequencing policy is examined by the simulation model.

2. Model presentation
In this section, the mathematical model is presented. This model is mostly similar to what is presented as a generic mathematical model in the book written by Sarin and Jaiprakash [7] (p. 33-60). In this model, the size of a sub-lot is equal through the production line (consistent sub-lot sizes), but the sizes of other products’ sub-lots may be different. A mixed integer linear programming model is employed to tackle the lot-streaming and sequencing problem. The goal is to minimize the finish time of the last job of the last sub-lot on the last machine, which is called a Makespan. There is no constraint on the buffer sizes between two successive stages. The assumptions are as follows:
- Lot-attached setup times
- Intermittent idling
- Sub-lot process intermingling
- No-preemption
- No transfer time

Furthermore, each sub-lot is assumed as a new job in the system. According to the classification format presented in [7], this model is classified as mF/N/C/I/I/CV/C_{max}\{intermingling\}. The model is presented below with the list of notations first.

Sets, variables and parameters

I = Set of stages. So |I| is the number of stages.

J = Set of jobs. So |J| is the number of jobs.

K = Set of sub-lots. So |K| is the number of possible sub-lots for each job.

\(p_{i,j} = \) Process time for one unit of job \( j \in J \) on stage \( i \in I \).

\(S_e_{i,j} = \) Setup time for job \( j \in J \) on stage \( i \in I \).

\(a_j = \) The lot size for job \( j \in J \).

\(s_{i,j,k} = \) The processing start time for sub-lot \( k \in K \) from job \( j \in J \) on stage \( i \in I \).
\( f_{i,j,k} \) = The processing finish time for sub-lot \( k \in K \) from job \( j \in J \) on stage \( i \in I \).

\( \theta_{i,j,k,j',k'} \) = A binary variable which is equal to 1 if the sub-lot \( k \in K \) from job \( j \in J \) on stage \( i \in I \) is processed before the sub-lot \( k' \in K \) from job \( j' \in J \).

\( b_{j,k} \) = The sub-lot size for the sub-lot \( k \in K \) from job \( j \in J \).

\( \beta_{j,k} \) = A control binary variable which is 1 if \( b_{j,k} \neq 0 \)

\( M = A \) sufficiently large number.

The objective function is to:

\[
\text{Minimize } \text{makespan} = \max_j \{ f_{i,j,k} \} \tag{1}
\]

Subject to:

\[
s_{i,j,k} + S e_{i,j} \beta_{j,k} + b_{j,k} p_{i,j} = f_{i,j,k}, \forall i \in I, j \in J, k \in K \tag{2}
\]

\[
s_{i,j,k} \geq f_{i,j,k-1}, \forall i \in I, j \in J, k \in K \tag{3}
\]

\[
s_{i,j,k} \geq f_{i-1,j,k}, \forall i \in I, j \in J, k \in K \tag{4}
\]

\[
\theta_{i,j,k,j',k'} + \theta_{i,j,k',j,k'} = 1, \forall i \in I, j \in J, k \in K \mid j \neq j' \text{ or } k \neq k' \tag{5}
\]

\[
s_{i,j,k'} - f_{i,j,k} \geq (\theta_{i,j,k,j,k'} - 1)M, \forall i \in I, j \in J, k \in K \mid j \neq j' \text{ or } k \neq k' \tag{6}
\]

\[
s_{i,j,k'} - f_{i,j,k} \leq \theta_{i,j,k,j,k'}M - \epsilon, \forall i \in I, j \in J, k \in K \mid j \neq j' \text{ or } k \neq k' \tag{7}
\]

\[
a_j = \sum_{k=1}^{K} b_{j,k}, \forall j \in J \tag{8}
\]

\[
b_{j,k} \leq \beta_{j,k} M \tag{9}
\]

\[
s_{i,j,k}, f_{i,j,k}, b_{j,k} \geq 0, \forall i \in I, j \in J, k \in K \tag{10}
\]

\[
\beta_{j,k} \in \{0,1\}, \theta_{i,j,k,j,k'} \in \{0,1\} \tag{11}
\]

Equation (1) is the objective function which should be minimized. The finish time of the last batch of the last job on the last machine is the makespan. The difference between the start time and the finish time of a sub-lot is equal to the setup time plus the processing time, which is stipulated in constraints (2). It is also called Sequential Processing Constraint in the literature. Logically, before processing a sub-lot on a machine, the previous sub-lot from that job must be finished (the first sub-lot of a certain job should be processed before the second sub-lot). Further, one sub-lot can be started to be processed on stage \( i \in I \) if the process on that sub-lot on the previous machine is finished. These logics are stipulated in constraints (3) and (4). The capacity of stages is one sub-lot at a time. Hence, one sub-lot must be finished before another sub-lot starts to be processed. Constraints (5), (6) and (7) ensure about the addressed logic. Constraints (6) and (7) care about this fact that if the start time of a sub-lot in a stage is greater than the finish time of another sub-lot on that stage, the binary variable \( \theta \) must be equal to 1; otherwise, it must be zero. Substituting equation (2) into constraints (6) and (7) results in a set of constraints known as intermingling constraints in the literature. On the other hand, processing of a sub-lot is either before or after processing another sub-lot, which is stipulated in (5). The summation of the sub-lot sizes of a job should be equal to the original lot size of that job (constraints (8)). In constraint (2), the setup time is included in the calculation of the finish time if \( \beta_{j,k} \) is equal to 1. \( \beta_{j,k} \) must be equal to 1, if \( b_{j,k} \neq 0 \). Constraint (9) considers the addressed conditions.
The presented model is applied to address the aforementioned analyses in the next section. Having the near-to-optimal solution from the model and the conclusions made for general problems, then, a batch-splitting policy is proposed in order to achieve a reasonably good makespan for a company. This is explained in the next section. The proposed policy is examined by a simulation model.

3. Case Study and Comparative Study
In this section, we aim at improving the performance measure of a company called Delta. Delta is a leading company in producing extruded, fabricated and anodized aluminum parts. Delta is a fictitious name due to confidentiality reasons.

3.1 Comparative Study
In the literature of scheduling and lot-streaming, many mathematical models are presented and heuristic methods are proposed. However, company managers and engineers rarely bother adopting models and applying heuristics to find the daily sequence. Therefore, an easy-to-implement policy should be introduced to workers and managers. Among the numerous products made by Delta, 5 frequently demanded products are selected. These products, with the amounts of demand they have, are considered as the representatives for the regular production process of Delta. All of the products, whose names do not appear in this article for confidentiality reasons, require to be processed through three stages. The codes, the demands, and the processing times of the products are reported in Table 1. These times are provided by the company that assumes fixed processing times. Furthermore, although production process is not performed one unit at a time, the unit process times are considered as an approximation for the batch production times, disregarding the batch size. It is worth noting that Delta considers unit processing time in its production planning as well. The production times are reported in Table 1.

<table>
<thead>
<tr>
<th>Job</th>
<th>Processing time in Workstation 1</th>
<th>Processing time in Workstation 2</th>
<th>Processing time in Workstation 3</th>
<th>Lot size</th>
</tr>
</thead>
<tbody>
<tr>
<td>7978</td>
<td>0.09 min</td>
<td>0.72 min</td>
<td>0.09 min</td>
<td>414</td>
</tr>
<tr>
<td>7977</td>
<td>0.07 min</td>
<td>0.42 min</td>
<td>0.05 min</td>
<td>1708</td>
</tr>
<tr>
<td>7808</td>
<td>0.10 min</td>
<td>0.75 min</td>
<td>0.24 min</td>
<td>836</td>
</tr>
<tr>
<td>7595</td>
<td>0.08 min</td>
<td>0.85 min</td>
<td>0.27 min</td>
<td>934</td>
</tr>
<tr>
<td>7596</td>
<td>0.08 min</td>
<td>0.75 min</td>
<td>0.24 min</td>
<td>836</td>
</tr>
</tbody>
</table>

Limited by the number of transportation facilities and rack sizes, each lot cannot be divided to more than 3 sub-lots. Due to the reasons mentioned above and this fact that solving a scheduling and lot-streaming problem for flow shop with 5 jobs and 3 stages may take a considerable time, it is proposed that company may split each lot into \(|K|\) (\(|K|\)=2 or 3) equal-sized sub-lots and apply SPT as the dispatching rule for the first stage and FIFO for the other stages (no-idling time). According to Baker and Jia [8], for three stage lot-streaming problems when stage two is dominant (has the largest processing time) the optimal makespan for the models assuming consistent sub-lots and equal sub-lots does not have a significant difference. Furthermore, under certain assumptions it is showed that equal sub-lot sizes are optimal where the sub-lot sizes are assumed to be consistent, such as the system studied in [9]. Besides, since SPT is a helpful dispatching rule to minimize makespan, the applied dispatching rule for the first stage is SPT followed by FIFO for the other stages. Thus, to split each lot to equal-sized sub-lots and apply SPT as the dispatching rule for the first stage may be helpful. This way, the people responsible for production planning are no longer required to build mathematical model and solve it to have the best values for the sub-lot sizes and the
respective sequences. Therefore, regarding that solving an MIP model like the model addressed in this article takes a lot of time and operational decisions require to be made immediately, a simple policy seems to be more practical.

To examine this proposed policy and to see whether this policy can generally reduce makespan considerably or not, a comparative study is conducted for general flowshop models. In this study, we aim at investigating the effect of number of sub-lots and sub-lot sizes on makespan under certain assumptions. By this comparative study it would be found that if any significant difference between the optimal makespan values in these three scenarios should be expected: 1) sub-lots are consistent, 2) sub-lots are equal sized and, 3) sub-lots are equal-sized and SPT would be applied to sequence the sub-lots on the first station and FIFO would be applied to sequence the sub-lots on the other stations. The first two scenarios are implemented by a MIP model and the third one on a simulation model built in ProModel 8.6.

In this case study, there are 3 stages and 5 jobs. However, since the computation time would be considerable to solve a scheduling and lot-streaming MIP model for this size, we considered a flowshop model with 3 stages and 3 jobs. Having the results from 3-stage and 3-job model, conclusions would be made and extended for a flowshop with 3 stages and 5 jobs which is the case for Delta. Setup times are uniformly distributed and process times are assumed to follow exponential distributions. According to Table 1, the process time of the second stage is dominant. Thus, the process times are randomly generated the way to have higher process times in stage 2. To catch uncertainty in process times and setup times, each scenario should be replicated a sufficient number of times for a given number of sub-lots for both MIP model and the simulation model. The sufficient number of replications would be achieved at the point that the average makespan value shows being statistically stable. The level of significance is equal to 5% in this research.

For the simulation model, however, the warm-up period should be obtained as the first step, prior to running the model for a sufficient number of replications. Generally, to obtain makespan value the simulation models are run for a long period of time to catch the transient conditions (long run). In this research, the transient conditions are caught by applying batch mean method. Considering 13,000 warm-up period, results for 3-stage 3-process flow shop model are obtained reported in Table 2.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Consistent sub-lots</th>
<th>Equal sub-lots</th>
<th>Equal sub-lots with SPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>[K]=1</td>
<td>Lower Bound</td>
<td>1449</td>
<td>2163</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1584</td>
<td>2200</td>
</tr>
<tr>
<td></td>
<td>Upper Bound</td>
<td>1719</td>
<td>2238</td>
</tr>
<tr>
<td>[K]=2</td>
<td>Lower Bound</td>
<td>1488</td>
<td>1539</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1627</td>
<td>1593</td>
</tr>
<tr>
<td></td>
<td>Upper Bound</td>
<td>1767</td>
<td>1681</td>
</tr>
<tr>
<td>[K]=3</td>
<td>Lower Bound</td>
<td>1449</td>
<td>1499</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1584</td>
<td>1636</td>
</tr>
<tr>
<td></td>
<td>Upper Bound</td>
<td>1719</td>
<td>1773</td>
</tr>
</tbody>
</table>

According to the results shown in Table 2, although the average values for equal-sized sub-lots and consistent-sized sub-lots are different, but there is no significant difference between them regarding 95% confidence interval. For equal-sized sub-lots, [K]=2 and [K]=3 confidence intervals for these cases overlap, concluding that increasing the number of sub-lots from 2 to 3 does not have a statistically significant difference on makespan. However, three sub-lots may be preferable considering the average values. Similar conclusions can be made for consistent sub-lot size model. For the proposed policy, makespan can be decreased by increasing the number of sub-lots. Furthermore,
comparing the confidence intervals obtained from the MIP model and the simulation model, significant different in the makespan can be concluded.

3.2 Case study
To examine the proposed policy for the case study, the setup times and processing times reported by the company are replaced by the randomly generated data. While Delta considers fixed processing and setup times, in this study the processing times with an exponential distribution with the average times mentioned in Table 1 are considered and seem more representative to the operations of the company. According to the conclusions made on the basis of information shown in Table 2, although the sequence obtained from solving the mathematical model is better than the proposed policy, implementing lot-streaming policy can significantly reduce makespan, as they do not currently implement any lot-streaming policy. Furthermore, since the mathematical model considers the empty system, the optimal value of makespan obtained by the mathematical model is the minimum possible makespan for the system, not necessarily the real one. To find the extent to which makespan can be reduced by the proposed policy, the impact of the number of sub-lots on the makespan is investigated. Setup times are also considered to be uniformly distributed from 5 minutes to 10 minutes. Warm-up period and number of replications are obtained for this case in a similar way as done for the generic case. Note that, the batch transfer times are included in the setup times to keep the inclusion of real situations in the simulation model. Increasing the number of sub-lots results in the makespan values presented in Table 3.

Table 3: The impact of number of sub-lots on the makespan with the proposed policy for Delta (5 products and 3 stages)

<table>
<thead>
<tr>
<th>Number of Sub-lots</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Makespan (min)</td>
<td>3202</td>
<td>3138</td>
<td>3172</td>
<td>3224</td>
<td>3242</td>
</tr>
</tbody>
</table>

Two important conclusions can be made from this figure. First, although the comparative study confirms improvement in makespan by increasing the number of sub-lots, these results show that increasing number of sub-lots does not necessarily reduce the makespan for this case study. After the point related to |K|=2, the makespan starts to deteriorate. This may happen due to the dominance of setup time on the sub-lot processing time where the size of sub-lots decreases. The two important parameters changed in the case study. These are the intervals and proportion of processing and setup times. Hence, the intervals of processing times and setup times’ variation and their relative proportion are the two parameters which may affect the optimum lot-streaming and sequencing policy. Second, the addressed technical and facility limitations to move sub-lots between machines do not affect the performance of the production system.

Since Delta currently has no lot-streaming policy and applies FIFO rule to sequence the jobs, from Table 2, improvement is expected to the system by applying the proposed policy. According to the results obtained by the simulation model with |K|=1 and FIFO dispatching rule, makespan is expected to be improved by 5% by implementing the addressed policy. Therefore, this policy is efficient in the current Delta’s conditions and this lot-streaming program can be implemented instead of solving the complicated model either by commercial and expensive modeling software or complicated heuristic and meta-heuristic methods.

To examine the quality of this solution, the MIP model is solved for |K|=2 and results for equal-sub-lots are compared with 3138 minutes as the best achievable makespan for Delta by implementing the proposed policy. Since it is proved that in a three stage flowshop lot-streaming and scheduling problem where the second stage is dominant, there is no significant difference between optimal makespan values for consistent sub-lot models and equal sub-lot models, only the result of equal sub-lot model is compared with the result of implementing the proposed plan. After solving the model for a sufficient number of times, the average value of makespan was found to be equal to 2821 minutes. Hence, it can be concluded that the sequence obtained by solving the optimization model can improve the makespan by 10% comparing with SPT.
To this end:

- Important conclusions are made about applying the addressed policy at Delta.
- The improvement the proposed policy examined by simulation can make in the makespan is calculated
- And potential capacity of improvement by applying adequate sequencing policy is introduced.

Next step is to further investigating of the problem to find what it would happen if the current conditions at Delta change. First, we suppose that through the implementation of a new technology, the setup times can be reduced. The impact of number of sub-lots on makespan assuming setup times which are reduced by 10, 20 and 30 percent is illustrated in Figure 1.

![Figure 1: The effect of reducing setup times on makespan and the required number of sub-lots for Delta case (5 products and 3 stages)](image)

According to the results shown in Figure 1, although decreasing the setup times results in decreasing the makespan, as expected, but still increasing the number of sub-lots makes the makespan to increase and the optimal value for the number of sub-lots is still 2.

What it would happen if the engineers find a method to reduce the process times? To find the answer, the effect of processing time for the dominant stage on makespan is investigated. Reducing the process time of the second stage by 10, 20 and 30%, the changes in makespan are illustrated in Figure 2.

![Figure 2: The impact of process time of the second stage on makespan and the required number of sub-lots for Delta case (5 products and 3 stages)](image)
According to Figure 2, even small reductions in the processing time of the second stage can reduce the makespan dramatically, as the second stage is the bottleneck. Decreasing the processing time by 10%, obviously results in an approximately 10% reduction in makespan. Yet, reducing the processing time of the second stage by 10% did not change optimal value of the number of sub-lots. However, as the dominant stage’s process time reduces by 20% or 30% while the setup times are fixed, the optimal sub-lot number changes to 3. This happens due to changing the proportion of process time of sub-lots to the setup time. Although reducing the process time of the second stage affected the average value of the makespan, these values are not statistically different considering 95% confidence interval. Totally, the results depicted in Figures 3 and 4 show that the proposed lot-streaming policy to split lot into two equal sub-lots is efficient and reliable whether Delta remains in its current practices or improves the setup times and processing times by 30%.

4. Summary and Conclusion

In this article, we aimed at finding a practical lot-streaming and scheduling policy to reduce makespan. We proposed a policy such that all jobs should be divided to equal sub-lots and then be sequenced on the first machine on the basis of SPT dispatching rule.

Applying a comparative study, the following conclusions have been made:

- Splitting the batches into equal sub-lots and applying SPT policy can also make significant reductions in makespan.
- There is no considerable difference between the makespan values where the consistent sub-lot sizes are equal and where they are not equal.
- Generally, the optimal sequence found by solving the scheduling MIP models is significantly better than SPT dispatching rule where minimizing makespan is the objective function.

The proposed policy investigated the input data received from Delta. Results show that Delta can reduce the makespan by applying the proposed policy by approximately 5%. However, the optimum makespan obtained by the MIP model is considerably better than what is achieved by the proposed policy. However, as mentioned earlier in this article, since operational decisions sometimes should be made immediately and optimizing the scheduling and lot-streaming model even for a mid-size shop floor takes a considerable time, a simple policy is required. The other important conclusion was that the limitations caused by transportation facilities, racking capacities and technical considerations have no effect on the performance measure of Delta production system given the current conditions.

According to the literature, scheduling and lot-streaming problems are placed in NP-hard category. Finding the makespan for a small problem with four products, three sub-lots and three stages may take a huge time to be found on a relatively powerful PC using the commercial optimization software. Especially in comparative studies which require investigating numerous scenarios and having numerous replications for each scenario, solution time may be so critical. Hence, extending this study for larger number of products and stages and having more replications to obtain more reliable solutions need High Performance Computers (HPC). Using these powerful computation tools, a more comprehensive research can be conducted in the future.

The reported conclusions are drawn on the basis of the general conditions with random setup and processing times. However, there may be some special cases in which these conclusions may not hold. Comparing the results obtained from the comparative study and case study, we claimed that the interval of variation and proportion of setup times and process time affect the optimum sequencing and lot-streaming policy. Another research study may be conducted to address these conditions.
References


