A Bivariate Real Options Model to Evaluate the Effects of Oil and Natural Gas Price Uncertainties on the Oil Sands Projects

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Abstract

This paper presents a practical, yet financially sound, real options framework to evaluate Canadian oil sands projects. The framework considers oil and natural gas price uncertainties, as well as managerial flexibilities in their decision making process. The Canadian oil sands hold the world’s third largest oil deposit. To process oil sands into a usable source of energy, an extensive amount of natural gas, which has a highly volatile price, is required. We show that the importance of natural gas price and its volatility on the optimal investment policies is fading, as the price has decreased significantly in recent years.

Keywords
OR in natural resources, Real options, Oil sands, Project valuation, Bivariate trinomial tree

1. Introduction

Canada ranks third, after Saudi Arabia and Venezuela, in terms of proven global crude oil reserves. Canadian proven oil sands reserves are estimated to be 170 billion barrels, located almost entirely in the province of Alberta in Western Canada [5]. To process oil sands into a usable source of energy, an extensive amount of natural gas is required. The substantial consumption of natural gas, a commodity with a highly volatile price, directly impacts the operating cost of an oil sands project. Over the last two decades, the price of natural gas has not followed a consistent trend. In 1993, the Henry Hub (HH) Gulf Coast natural gas spot price was approximately $2/MMBtu. The price soared more than seven times to $15/MMBtu in 2005, then decreased more than five times to its current level of $3/MMBtu. The increase resulted largely from the California energy crisis, cold weather, and hurricanes, while the later decrease resulted from the economic crisis of 2008, increased natural gas production, increased storage capacity, and slow economic recovery; both movements illustrate the long run volatility of this commodity. Figure 1 presents the historical trend of Henry Hub (HH) natural gas spot price and the West Texas Intermediate (WTI) oil spot price since 1990. Given the significant dependency of the oil sands industry on natural gas, the uncertainty in natural gas price may appear to be an important risk factor for the industry.

Besides concerns over natural gas consumption, environmental opposition, and labor constraints, the oil sands sector is now struggling with a new concern: limited logistics to transport the Canadian heavy crude oil to tidewater ports, where there is a greater demand for the product compared to the U.S. Midwest market, where the majority of the Canadian crude oil is currently being exported to. The objective of this work is to provide a sound, yet practical, real option model to evaluate oil sands projects in western Canada, while considering the uncertainty of the oil and natural gas prices. We apply our valuation model to two extreme price scenarios: scenario of year 2005, when oil prices were relatively low and natural gas prices were relatively high, and scenario of year 2013, when oil prices were relatively high and natural gas prices were relatively low.
In contrast with traditional valuation techniques such as net present value (NPV), which assume managerial discretion is limited in the face of uncertainty, real options analysis accounts for managerial flexibilities in responding to future events as uncertainty is resolved [12]. Although the inability of a classic NPV analysis to capture the value of management flexibility in a capital budgeting analysis is now well documented by Trigeorgis [12] and many others, industry surveys demonstrate that there is limited adoption of real options analysis within an applied setting due to its complexity and lack of intuition in the solution procedures [1]. This paper translates the theory and mathematics of real options analysis into a model that is understandable for managers and can be easily integrated into the current decision-making framework of the oil sands companies.

Numerous models have been proposed to estimate the value of an oil field, given market and technical uncertainties, using simulation and numerical techniques, such as Monte Carlo simulation and tree techniques (see for example [3]). Traditional Monte Carlo simulation techniques fail to handle the early exercise features of American options, and to solve this problem, Longstaff and Schwartz [8] proposed a novel technique called Least Square Monte Carlo (LSMC) simulation. Even though LSMC is applicable to almost any collection of early exercise features, even for fairly simple early exercise options, the optimal exercise boundaries can be noisy, despite the option value being fairly accurate. We believe that for the purpose of this study, with a significant number of compounded American options and different pricing scenarios, tree techniques are more suitable alternatives. Tree techniques allow practitioners to visualize different scenarios with multiple uncertainties and complex options, and are easy to understand and flexible. In this paper, we model the oil and natural gas prices as two one-factor mean-reverting stochastic variables, and evaluate the early exercise boundaries of a set of compound American real options, using bivariate trinomial tree techniques in an oil sands context. The rest of the paper is organized as follows: the Methodology Section explains our proposed real options model, the Results Section explains the findings of the paper, and the Conclusion Section summarizes these findings.

2. Methodology

There are well-known and compelling economic reasons supporting the idea of commodity prices following mean-reverting stochastic processes (for example see Eydeland and Wolyniec [4]). Consequently, we assume that oil price follows a one-factor geometric mean-reverting (GMR) process, which satisfies the stochastic differential equation

$$dS_t = \kappa(\theta(t) - \ln(S_t))S_t \, dt + \sigma S_t \, dW_t,$$  \hspace{1cm} (1)

where \(\theta(t)\) is the level of mean-reversion, \(\kappa\) is the rate of mean-reversion, \(\sigma\) is the volatility of the oil prices, and \(W_t\) is a standard Brownian motion under the risk-neutral measure. To incorporate an increasing/decreasing level of mean-reversion, we assume that \(\theta(t)\) is linear in time (\(\theta(t) = \theta_0 + \theta_1 t\), where \(\theta_0\) and \(\theta_1\) are constants). We calibrate the mean-reversion parameters of the spot process, \(S_t\), using the least squares approach developed by Hikspoors and Jaimungal [6], a method that produces similar results to Kalman filtering.

Since there is an extensive number of contracts written on the Western Texas Intermediate (WTI) crude oil, a bench-
mark for the U.S. Midwest market, WTI prices are usually used to estimate the Canadian crude oil prices (see for example \cite{10}). However, some adjustments are required to account for the WCS-WTI price differential that results from a lower quality of WCS compared to WTI, and the cost of transportation of WCS from western Canada to the U.S Midwest. We use monthly Cushing, Oklahoma, WTI oil futures contracts obtained from Bloomberg for the years 1990 to 2013 for our calibration. Once calibrated, we use the following equation to adjust the the spot value to account for the price differential

$$S_t' = \phi_1 S_t - \phi_2,$$

(2)

where $\phi_1$ is a constant between 0% and 100%, representing the quality difference, and $\phi_2$ is the transportation cost per barrel. According to industry experts, a $3 to $8/bbl cost of transportation may apply to transport Canadian crude from Alberta to the U.S Midwest, depending on such factors as the type of tariff contracts, the quality of the blend, and the distance between a plant and the refinery. Here, we assume a transportation cost of $5/bbl. Once adjusted for the transportation cost, the WCS/WTI ratio on average equals approximately 0.8, so we assume $\phi_1 = 0.8$.

Similarly, we model the spot natural gas price, $G_t$, as $G_t = \exp(f_t + g_t)$, where $f_t = A \sin(\omega t + B)$ is a seasonal factor (to account for the seasonal characteristics of natural gas prices), which is the amplitude of the seasonality component of the natural gas price and represents the peak deviation from the mean-reversion level at time $t$, $\omega$ is the angular frequency of the natural gas price, which specifies how many oscillations occur in a year, and $B$ is the phase which specifies where in the cycle the oscillation begins at $t = 0$. Also, $g_t$ is a one-factor GMR process, which satisfies:

$$dg_t = \eta[\alpha(t) - \ln(g_t)] g_t \, dt + \gamma g_t \, dZ_t,$$

(3)

where, $\alpha$ is the level of mean-reversion of the natural gas price, which we assume to be constant, $\eta$ is the rate of mean-reversion, $\gamma$ is the volatility of the natural gas price, and $Z_t$ is a standard Brownian motion under the risk-neutral measure, which is assumed to be uncorrelated to $W_t$. Note that although the correlation between the oil and natural gas spot prices was historically significant, the correlation had been negligible in recent years, and the trend is expected to continue in the future (see for example \cite{2}). An interested reader can refer to Hull and White \cite{7}, who modified the branching probabilities of the tree to account for a correlation between the two state variables.

The futures price $F_t^G(T) = \mathbb{E}[G_T]$ can be explicitly solved as

$$F_t^G(T) = \exp\left\{f(t) + e^{-\eta T} \ln G_t + \bar{\theta}_0 (1 - e^{-\eta T}) + \theta_1 \left(\tau - \frac{1 - e^{-\eta T}}{\eta}\right) + \frac{\gamma^2}{4\eta} (1 - e^{-2\eta T})\right\},$$

(4)

where $\bar{\alpha} = \alpha - \frac{\gamma^2}{2\eta}$ is the convexity adjusted long-run mean of the natural gas price. Similar to the oil mean-reversion parameters, we calibrate $A$, $\omega$, $\alpha$, $\eta$, and $\gamma$ to match the natural gas futures prices, using the least squares approach developed by Hikspoors and Jaimungal \cite{6}. We use the daily HH Gulf Coast natural gas futures contracts obtained from the Bloomberg dataset for each case study. Table \ref{table1} in the Results Section summarizes the calibrated mean-reversion parameters of the oil and natural gas price models for each case study.

In the Results Section, we apply different natural gas price volatilities, $\gamma$, to estimate the critical construction boundaries, using our bivariate model, and compare the results. It should be noted that investigating the role that differing levels of natural gas price volatility have on the critical construction boundary cannot be carried out by simply using Equation (4) with various volatility values. The limitation to this approach is that, with all else kept constant, increasing (or decreasing) volatility will produce futures curves whose expected paths are different from the base case given by the estimated mean-reversion parameters, leading to biased comparisons. Instead, we opt to slightly modify the drift of the natural gas price model such that the expected futures curves for all volatility levels are identical. Specifically, we write the adjusted oil price model as $\tilde{S}_t = e^{h(t)} S_t$, with $S_t$ still satisfying Equation (1) with an arbitrary volatility level, and

$$h(t) = \left\{\frac{1}{4\kappa}(1 - e^{-2\omega t}) - \frac{1}{2\kappa}(1 - e^{-\omega t})\right\}\left(\sigma^{(0)} - \sigma^2\right).$$

(5)

In this manner, $\tilde{S}_t$ is a modification to the deterministic portion of oil prices such that the the futures prices of $\tilde{S}_t$, are equal to the futures prices from the original model with a volatility of $\sigma^{(0)}$ – the base volatility level. Specifically using the equation for futures prices obtained from Equation (1) we have

$$F_t^{\tilde{S}}(T) = \exp\left\{h(t) + e^{-\kappa T} \ln S_t + \bar{\theta}_0 (1 - e^{-\kappa T}) + \theta_1 \left(\tau - \frac{1 - e^{-\kappa T}}{\kappa}\right) + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa T})\right\},$$

(6)

$$= \exp\left\{e^{-\kappa t} \ln S_t + \bar{\theta}_0 (1 - e^{-\kappa t}) + \theta_1 \left(\tau - \frac{1 - e^{-\kappa t}}{\kappa}\right) + \frac{(\sigma^{(0)})^2}{4\kappa} (1 - e^{-2\kappa t})\right\},$$

(6)
where $\tilde{\theta_0} = \theta_0 - \frac{(\sigma(0))^2}{2\kappa}$. Hence, regardless of the volatility of the modified spot price, $\sigma$, the implied futures curves are all identical and can be calculated by the futures curve with a base (calibrated) volatility of $\sigma(0)$.

To calculate the value of the cash flow ($CF$) on each node of the tree, we have,

$$CF = S' - n \times G - FC,$$

where $S'$ is the oil spot price and $G$ is the equivalent natural gas cost, and $FC$ is an average fixed cost that accounts for royalties, income tax, emission compliance costs, abandonment costs and all other operating costs. While the breakdown of the fixed costs can vary significantly from one project to another, depending on the method of bitumen recovery, the size of the plant, the quality of the reserves, etc, Millington et al. [9] reports an average breakdown for the cost of production of crude bitumen, using steam-assisted gravity drainage (SAGD) and surface mining. By using Millington et al. [9]’s estimates and consulting with industry experts, we assumed an average value for each component of the $FC$, which is reported in the Results Section.

In our real options model, we assume management, i.e., an agent, has the option to construct an oil sands plant, $P$, or wait to invest in the next time-step, i.e., $t + \Delta t$. We further assume that the agent can forecast the future cash flows a priori for the next period only and thus can permanently shut down a plant before experiencing negative cash flows. Note that the plant may still run during negative cash flows because the expected future value may be greater than the immediate negative cash flow. In a general production project, an active project will be mothballed when conditions become sufficiently adverse and an idle project will be reactivated when prices are high enough (i.e., when demand conditions become sufficiently favorable). However, according to industry experts, plants are either activated or completely shut down. Mothballing is not a typical practice in oil sands projects. This assumption simplifies the analysis.

The option to construct the plant is an early exercise compound option with the additional option of shutting the plant down and receiving (or paying) a salvage if it is no longer profitable to run the plant. The standard dynamic programming principle implies that these two sequential compound options must be valued in a backward manner: the second chronological option (the option to operate or shut down) must be considered as the risky underlying asset of the first chronological option (the option to construct or delay) and must be calculated first.

To begin valuing these real options, we introduce a trinomial tree for the option to operate plant $P$, denoted by $P^O$. Each node of this tree represents the value of $P$ at a given time and a given oil price. The superscript $O$ refers to the operational flexibility. We set the values of the terminal nodes to the salvage value of $P$, and the calculation proceeds in a backward manner from the final time-step to the first. At each step the option to shut down requires comparing the following two values: the value of continuing the operation and receiving the cash flow of the present time-step plus the discounted expectation of the future value, versus the value of shutting the plant down and receiving the salvage value.

Since the agent may choose between running the plant and shutting it down, the value at any given node is the maximum of these values. The backwards recursion for $P$ is as follows:

$$P^O_t = \max \left( (S'_t - G_t - OC) + e^{-r_f \Delta t} \mathbb{E}^Q \left[ P^O_{t+\Delta t} \right] , SV \right),$$

where $P^O_t$ represents the values of lattice $P^O$ at time $t$, the term $(S'_t - G_t - OC)$ is the operating cash flow, while running during the next time interval, term $\mathbb{E}^Q \left[ P^O_{t+\Delta t} \right]$ is the expected value of the future cash flows of $P$, $r_f$ is the risk-free rate, and $SV$ is the salvage value. Whenever the optimal cash flow plus the discounted expectation is less than the shutdown value, $SV$, the plant is closed and cannot be turned on again. This calculation determines the optimal shutdown policy – also called the critical shutdown boundary.

The next option to evaluate is the option to construct $P$. Upon exercising this option, the agent’s remaining decision is how to optimally operate the plant. Consequently, the lattice $P^O$ (obtained in the previous stage) acts as the underlying asset for the option to construct $P$. Let $P^C$ denote the lattice corresponding to the optimal construction of $P$. At the end of the lease contract, there is no more value in the option to construct, so the terminal nodes are set to zero and we proceed backward to obtain the value of the option to construct at earlier times. The option to construct provides the agent at each time-step with two possibilities: a) paying the construction cost, constructing $P$ at the present time-step, and receiving the discounted expected cash flows (subject to operational flexibility), or b) delaying construction, which has a value equal to the discounted expectation of the option to construct $P$ in the next time-step. The boundary along
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Table 1: Mean-reversion parameters of oil and natural gas prices.

<table>
<thead>
<tr>
<th>Case</th>
<th>Date of Study</th>
<th>Monthly futures contract inception</th>
<th>Oil spot at the date of study</th>
<th>Oil MR parameters</th>
<th>NG Spot at the date of study</th>
<th>NG MR parameters</th>
<th># of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>J-95</td>
<td>1990-95</td>
<td>15.30</td>
<td>0.05</td>
<td>0.21</td>
<td>0.54</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>J-96</td>
<td>1991-96</td>
<td>18.20</td>
<td>0.45</td>
<td>0.30</td>
<td>0.10</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>J-97</td>
<td>1992-97</td>
<td>17.30</td>
<td>0.30</td>
<td>0.45</td>
<td>0.10</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>J-98</td>
<td>1993-98</td>
<td>24.30</td>
<td>0.05</td>
<td>0.21</td>
<td>0.54</td>
<td>40</td>
</tr>
</tbody>
</table>

which the maximum is attained by the first term is the critical construction boundary for $P_C$.

$$P_C^* = \max \left( -CC + e^{-r(\Delta t)}E_Q \left[ P_C^0 \right] - e^{-r(\Delta t)}E_Q \left[ P_C^{i+\Delta t} \right], 0 \right).$$

where, $\Delta CP$ is the construction period and $CC$ is the construction cost. The boundary along which the maximum is attained by the first term is the critical construction boundary for $P_C$. Note that the option to construct $P_C$ will expire by the end of the lease term, $T$, minus the construction period ($T - \Delta CP$), because if construction does not begin by this point there will not be enough time to complete the plant and receive a cash flow.

3. Results and discussions

In this section, we consider 19 cases, where each case assumes a hypothetical oil sands plant, $P$, that is values using the model above, as of year $\tau_i$, $\tau_1 = 1995$, $\tau_2 = 1996$, ..., $\tau_{19} = 2013$. For any given year, for example year 1995, we calibrate the mean-reversion parameters of the oil and natural gas prices based on the future prices that were available in that year[4], calculate the real options value of plant $P$, using the calibrated parameters, and compare the trend of this value over time with the historical trend of the oil sands development in Alberta. To investigate the impact of the parameters on the value of $P$, we consider two particular price scenarios in Cases 11 and 19, and perform a sensitivity analysis on the estimated parameters in each case. Case 11 presents the price scenario of year 2005, when the natural gas price was relatively high and the oil price was relatively low, and Case 19 presents the price scenario of year 2013, when the natural gas price was relatively low and the oil price was relatively high. Moreover, we calculate the critical oil and natural gas prices to invest in plant $P$ and to operate it, and provide a sensitivity analysis on the volatilities of the natural gas price, to see its impact on the critical construction boundary, in both scenarios.

We assume that the life of plant $P$ is 35 years and its reserves will not run out during this period. We also assume that the plant produces WCS using mining techniques, and we use Millington et al. [9]'s cost estimates to account for the capital and operating expenditures. We assume a risk-free rate of 5% and an inflation rate of 2% for the costs. Table 1 presents the mean-reversion parameters of the 19 cases.

Figure 2 presents the real options value of $P$ in the last two decades, as well as the number of Alberta oil sands projects that have started their front end engineering design (FEED) phase and been approved for construction. We consider this number to be a proxy for the oil sands development in Alberta. Figure 2 also shows the construction decision that is made based on our model. As shown, there is a positive correlation between the trend of the real options value over time and the number of the plants approved for the construction: they have both significantly increased. This development can be explained by the significant increase in the oil price and the significant decrease in the natural gas price during the last few years.

\[1\] In this example, we used the futures contracts that were issued during January 1990 to December 1995
Figure 2: The real options value of plant \( P \) in the last two decades, using the parameters of Table 1 and the number of Alberta oil sands projects that have started their front end engineering design (FEED) phase and been approved for construction.

Figure 3: Futures curves of the oil and natural gas prices, using Equations (1) and (3). Figure 3 presents the futures curves of the oil and natural gas prices, obtained by solving Equations (1) and (3). Sub-Figure 3(a) presents Case 19, and Sub-Figure 3(b) presents Case 11.

Figure 4 presents the sensitivity of the value of plant \( P \) to the long-run mean of the oil and natural gas prices (\( \exp(\theta_0) \) and \( \exp(\alpha) \) in Equations (1) and (3)), in Case 19 (left panel) and Case 11 (right panel). When comparing the two curves in Sub-Figure 4(a), we realize that in Case 19 the value of the plant was significantly sensitive to \( \theta_0 \), but insensitive to \( \alpha \). This can be explained by the magnitude of the gas price compared to the oil price, in Case 19 (i.e., the current market). Even a 50% increase in the mean-reversion level of the natural gas price barely impacts the value of the project, whereas only a 10% decrease in the mean-reversion level of the oil price reduces the value by 18% (approximately $1 billion in this example). However, this was not the case in 2005. As shown in Sub-Figure 4(b), a 50% increase in the mean-reversion level of the natural gas price in Case 11 decreases the value of the plant by 20%. Our model estimates that the value of plant \( P \) in Case 11 is approximately $0.3 billion, which is substantially lower than the value in Case 19 ($5.5 billion), mainly due to a lower oil price (and also due to a higher natural gas price). Therefore, the value of the plant in Case 11 is more sensitive to the long-run levels of both oil and natural gas prices than that in Case 19. Therefore, a change in the long-run level of the natural gas price can significantly impact the decision making process.
Similarly, Figure 5 presents the sensitivity of the value of $P$ to the volatility of the oil and natural gas prices ($\sigma$ and $\gamma$), in Case 19 (left panel) and Case 11 (right panel). Comparing the curves in Sub-Figure 5(a) reveals that the value of the plant is significantly sensitive to the volatility of the oil price, but insensitive to the volatility of the natural gas price at any reasonable $\gamma$, in Case 19. Even a 100% increase in $\gamma$ barely changes the project value, which again emphasizes the fading importance of the natural gas price uncertainty in the valuation process, in the current market. Again, we can see in Sub-Figure 5(b) that this was not the case in 2005, when the volatility of the natural gas price could have had a significant impact on the value of the project. Figure 5 also demonstrates that as the volatility of the oil price (revenue) increases, the value of the plant improves. This can be explained by the classic characteristic of real options, which is their asymmetric, or kinked, payoff. Since managers have the option to terminate the project in an unfavorable market condition, downside risks are mitigated. In retrospect, the upside potentials are maximized such that the returns are increased, since the project will only be executed if the market condition is encouraging.

A sensitivity analysis on the value of $P$ to the rates of mean-reversion of the oil and natural gas prices ($\kappa$ and $\eta$ in Equations (1) and (3)) also shows that in Case 19 the value of the plant is significantly sensitive to $\kappa$, but not sensitive to $\eta$ at any reasonable level; whereas in Case 11 the value was significantly sensitive to both $\kappa$ and $\eta$.

Figure 5 illustrates the critical operating and construction boundaries of plant $P$, using the parameters of Case 19. The operating boundary indicates the critical spot prices for oil (WCS) and natural gas (HH) to operate the "on-stream" (constructed) plant $P$ or to shut it down, at each time-step. Once the spot oil price is above the critical oil price and the spot natural gas price is below the critical natural gas price, the agent will operate the plant, and once reversed, the
Figure 6: Critical boundaries of $P$, using the oil and natural gas price parameters of Case 19 (year 2013).

As shown in Sub-Figure 6(a), the operating boundary increases as we move forward in time. This observation can be explained by the value of the option to operate the plant. When the option reaches maturity, the critical operating boundary approaches the total cost of $TC = G + OC$. At this time, if the value extracted from the oil sales is higher than the total cost, the cash flow will be positive. However, as we move backward in time from the maturity, there is an additional value in the option to operate the plant, even though the total cost may exceed the value extracted from the oil sales. Therefore, a plant may continue running, despite of receiving negative cash flows because the potential future cash flows may be large enough to offset the immediate negative cash flows. Using the estimated parameters of Case 19, the futures oil prices are expected to be just above $300/bbl in 35 years, which means that the future value of running the plant is higher than the value of shutting down the plant and receiving the salvage value immediately. Therefore, the critical operating boundary moves downward as time goes backward from maturity.

Sub-Figure 6(a) also demonstrates that, the critical spot oil price to shut down the plant is significantly low, for approximately the first 30 years, compared to the current spot oil price. As the plant approaches maturity, the critical spot price increases. This observation implies that once the plant is constructed and the construction cost is sunk, the oil price needs to drop substantially for the plant to shut down. In the current bullish oil market and with the long life of oil sands projects, the value of the expected future cash flows outweighs the salvage value and operating costs, and an oil sands plant is unlikely to close any time soon. This result is in line with the recent report from London-based consultants Wood Mackenzie, which states that on-steam projects with a significant amount of capital expenditure sunk are unlikely to be shut down as a result of the current volatile and discounted oil prices, although their economics can be impacted [11].

A comparison between Sub-Figures 6(a) and 6(b) also reveals that the critical construction boundary is much higher than the critical operating boundary. This observation indicates that, unlike the decisions regarding terminating an on-stream project, the decisions regarding the construction of an unconstructed (unsanctioned) project are significantly sensitive to the oil price. In Case 19 and at time $t = 0$, the critical spot oil price to construct plant $P$ is approximately $50/bbl$. Considering the WCS prices in the last few years, we understand that the margin between the current and the critical spot oil prices to construct the plant is quite narrow and that unsanctioned projects with little or no capital spent are susceptible to delay or even cancellation. The Syncrude Stage 4 and the Total Joslyn projects are examples of unsanctioned projects for which construction has been postponed; and projects such as Horizon, Kai Kos Dehseh’s Corner phase, and Narrows Lake are examples of the projects that are prone to delay [11]. Figure 6 also shows that as time progresses, the construction boundary increases, indicating that the amount of time left to receive cash flows decreases, which in turn decreases the value of the option to wait. When the option to construct expires in year 32, the boundaries increase to infinity.

Figure 7 illustrates the critical operating and construction boundaries of plant $P$ using the mean-reversion parameters of Case 11. When comparing Sub-Figures 7(a) and 6(a) we realize that the critical spot oil price to shut down the
plant in Case 11 is higher than that in Case 19. Sub-Figure 7(a) also reveals that the critical operating boundary first decreases and then increases, as we move backward in time. Similar to Case 19, when the option reaches maturity, the critical operating boundary approaches the total cost of $TC$. However, as we move backward in time from the maturity, the potential future cash flows are lower than those in Case 19 in order to offset the immediate negative cash flows (see Figure 3). Thus, the critical spot prices to shut down the plant are higher. Similar to Case 19, in Case 11 the critical construction boundary is significantly higher than the critical operating boundary, implying that once the plant is constructed the oil price needs to drop substantially for the plant to shut down.

When comparing Sub-Figures 7(b) and 6(b), we realize that unlike the critical construction boundary of Case 19, the boundary of Case 11 is decreasing, as we move forward in time and the natural gas price follows its expected path shown in Figure 3. This observation implies that when the highly uncertain natural gas price is expected to decrease, there is more value to wait and obtain more information than to immediately invest (this result is also shown in Figure 2).

To investigate the impact of the natural gas price uncertainty on the decisions regarding the construction of an unsanctioned plant, we perform a sensitivity analysis on $\gamma$, the volatility of the natural gas prices in Equation 3, in Cases 11 and 19. Figure 8 shows the critical construction boundaries of $P$, when natural gas price follows its futures curve, shown in Figure 3, and $\gamma = 0, 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0$. Sub-Figure 8(a) presents Case 19, and Sub-Figure 8(b) presents Case 11. As shown in Sub-Figure 8(a), the critical construction boundary of plant $P$ estimated by our bivariate model equals the critical boundary estimated by a univariate model (see the deterministic curve in Sub-Figure 8(a)). This observation implies that in the current market, with a natural gas price that is substantially lower than the oil price, a univariate model is sufficient for the valuation process. Conversely, Sub-Figure 8(b) reveals that as the volatility of the natural gas price increases to 45% or higher, the critical construction boundary of plant $P$ rises. This can be explained by the value of the option to postpone the investment, which increases as the uncertainty in the future operating cost increases. In other words, at a higher $\gamma$, a higher critical spot oil price will be required to forgo the value of the waiting option. According to the Canadian Natural Gas Initiative, the natural gas price is expected to remain affordable; therefore the uncertainty of the natural gas price will no longer impact the development of the industry.

4. Conclusions
This article presents a practical real options framework for evaluating an undeveloped oil sands field. It considers two sources of uncertainty, oil and natural gas prices. Our model allows practitioners to visualize different scenarios while accounting for the options that are available to managers to alter their initial decisions as the uncertainties are resolved. Using our model, we conclude that oil price is a key driver of the oil sands growth in Alberta. Conversely, considering the current low natural gas price, the importance of natural gas price in the decision-making process is fading. However, this was not the case in 2005, when the natural gas price was significantly higher than today’s price; thus, considering the stochasticity of this price could have impacted the decision making process. We also conclude that once an oil sands plant is constructed and the construction cost is sunk, the oil price needs to drop drastically for the plant to shut down. In the current bullish oil market and with the long life of oil sands projects, the value of the
expected future cash flows outweighs the operating costs for at least the first 30 years from the construction of a plant, and an oil sands plant is highly unlikely to close any time soon. Unsanctioned projects with little or no capital spent, however, are susceptible to delay or even cancellation, considering the WCS prices in the last few years. The Syncrude Stage 4 project and the Total Joslyn project exemplify this point.

References