Pricing Bilateral Gas Contract for a GENCO

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Abstract

In Singapore, about 80% of electricity is generated from natural gas. With increasingly volatile gas prices, the profitability of a power generation company (GENCO) heavily relies on its ability to manage a natural gas portfolio. In this paper, we study a problem of pricing a bilateral natural gas contract for a GENCO. To evaluate the value of the contract, we consider the optimization problem of dynamically allocating contracted gas over a planning horizon taking into account volatile spot gas prices. The problem is formulated as a multistage Markov decision process (MDP). We show that a base stock policy is optimal and compute the optimal base stock levels by solving the MDP with approximate spot gas price scenarios. To determine the bilateral contract price and the contracted gas amount, we build a Nash bargaining equilibrium model by incorporating the values of the contracts for both the GENCO and the gas supplier. Numerical results under various market conditions are presented to demonstrate the feasibility of the model in practice.

Keywords

1. Introduction

For the last decades, a large number of energy markets in the world have been restructured and deregulated. It has been observed that energy fuel prices become increasingly volatile as energy markets are more liberalized and competitive. Power generation companies (GENCO) are exposed to more significant risks caused by fuel price fluctuations as well as demand uncertainties, and it is necessary for a GENCO to develop a plan for strategic fuel supply management to mitigate the risks and reap more fuel cost savings. Natural gas is considered as a promising source for power generation, because of its clean burning nature compared to other traditional power generation fuels. In the United State, electricity generated by natural gas consists of 15% of the nationwide electricity generation in 2000, and the number will rise up to 33% in 2020 [1]. Energy Market Authority in Singapore reports that approximately 80% of electricity supplied to national power grid is produced from natural gas in 2007 [1]. A survey from GENCO executives in Singapore shows that fuel cost accounts for approximately 70% of the operating cost for power generation [2]. In this paper, we consider a gas-fired GENCO that can engage in bilateral forward contracts to manage its natural gas supply. With gas prices becoming increasingly volatile, the profitability of the GENCO heavily relies on its ability to manage the gas supply portfolio over bilateral contracts and spot gas trading. One of the primary concerns of the GENCO is how to determine the price and the amount of gas engaged in a bilateral contract.

A verity of energy portfolio optimization problems have been extensively studied in many practice-based research papers. The problems are often modeled as a stochastic mixed integer linear programming problem and are solved by commercial software such as Cplex, without looking deep into the structure the model [3, 4]. Algorithms based on Lagrangian relaxation method and bender’s decomposition have been proposed to efficiently solve energy portfolio and generation planning problems by exploiting the structure of the optimization models [5]. Another popular stream of research in this area is to formulate the problems as a stochastic dynamic programming and develop an efficient algorithm by exploiting the property of the value function; see [6, 7]. Secomandi [8] studies a natural gas storage valuation problem in a capacitated setting, and establishes a two stage structure of an optimal base stock inventory trading policy. In this paper, we consider the optimization problem of dynamically allocating contracted gas over a finite planning horizon taking into account volatile spot gas prices and stochastic demands. We formulate the problem as a multistage Markov decision process (MDP) and show that a base stock policy is optimal.
In practice any operational schedule determined by one party probably would not be optimal for the other one. Hence, it is necessary to bilaterally negotiate the contract provisions, such as the total contract volume and contract price. Gabriel et al. [9] investigate a problem of determining the contract price and quantity for an electric retailer considering settlement risks on both the supplier and the end-user sides. Dong and Liu [10] study a problem of pricing a forward contract of nonstorable commodities for a risk-averse decision maker. Wu and Kleindorfer [11] investigate a portfolio optimization of forward contracts and spot trading for one buyer and multiple sellers, and establish a Bertrand-Nash Equilibrium for the contract price. In this paper, we study a problem of pricing bilateral gas contract for a risk neutral GENCO. The values of the contracts for both the GENCO and the gas supplier can be evaluated by solving MDP models, and a Nash bargaining model is proposed to determine the contract price and quantity. The unique Nash equilibrium solution implies that the optimal contract quantity and price can be determined separately in sequence in the negotiation process.

The remainder of this paper is organized as follows. The MDP formulation of a natural gas supply management problem for a GENCO is introduced in section 2. We investigate the characteristics of the value function and derive an optimal base stock policy. Section 3 proposes a mathematical model to evaluate contracts for the GENCO and the natural gas supplier. In section 4 we introduce a Nash bargaining model to determine the contract price and quantity and discuss the characteristics of the model. Section 5 presents numerical results under various market conditions and provides managerial insights. We draw the conclusions of this paper and outline future research directions in section 6.

2. Natural Gas Supply Management

In this section, we describe our natural gas supply management problem and model it as a stochastic dynamic programming (SDP) and show the convexity of the value function to derive an optimal base stock policy.

2.1 Problem Description and Model Formulation

Consider a natural gas supply and procurement problem for a GENCO in the presence of a gas spot market. To hedge against procurement risk and guarantee supply availability, the GENCO would engage into a bilateral forward contract. In general, the bilateral contract specifies the quality and quantity of the natural gas at a predetermined price and the requirements for delivery at specific times \( t \in \{1, 2, \ldots, T\} \) in the future. But the GENCO can determine the delivery amount at each delivery time under certain restrictions. The goal of the GENCO is to make full use of the contract in the face of stochastic spot gas prices and uncertain demands in the future.

![Figure 2.1: Description of the sequence of events](image)

The description of the sequence of events over a finite time planning horizon is presented in Figure 2.1. At the beginning of the time horizon, the GENCO decides the total delivery quantity \( Q \) and price \( J(Q) \) of the contract over the entire planning horizon. At the beginning of period \( t \), the remaining amount of contract natural gas \( x_t \) is updated and the natural gas spot price \( p_t \) for the time period \( t \) is revealed. In electricity markets, any committed power demand must be satisfied. The day ahead auction mechanism enables a power producer to adaptively adjust its electricity production schedule to match its daily demand. Therefore, we assume that the power demand \( D_t \) is known at beginning of period \( t \). For the model formulation, we represented \( D_t \) as the amount of natural gas required to meet the power demand for the period \( t \). Furthermore, the electricity demands \( D_t, t = 1, 2, \ldots, T \) are assumed to be independent of natural gas spot prices \( p_t, t = 1, 2, \ldots, T \). The evolution of natural gas spot price process is modeled by a single-factor mean-reversion model to capture the mean-reverting feature observed in natural gas spot market [13]. We denote the range of the remaining gas by \( X := [0, Q] \) and the range of the power demand by \( D := [D, D] \), where \( 0 \leq D < D \).

By incorporating the information on \( x_t, p_t \) and \( D_t \), the GENCO decides the amount of gas \( a_t \) to be withdrawn from the remaining contract gas for the period \( t \). The contract withdrawal amount \( a_t \) is limited by the capacity \( C_d \) of the
pipeline network and the available remaining amount of contract gas $x_t$. Due to non-storability of electricity, the delivery amount $a_t$ cannot exceed the power demand $D_t$, and thus the range of $a_t$ is defined by $\mathcal{A}_t = \{a_t \in \mathbb{R}_+: 0 \leq a_t \leq \min\{C_d, x_t, D_t\}\}$. In our problem, we only consider fuel costs, ignoring other operational costs and in-kind fuel losses. The action $a_t$ yields an immediate cost of purchasing $(D_t - a_t)$ amount of gas from the spot market, which is expressed as $c_t(a_t, p_t, D_t) = p_t(D_t - a_t)$.

Our objective is to minimize the expected total spot gas procurement cost over the whole planning horizon, provided that the initial state $(x_1, p_1, D_1)$ at the beginning of the time horizon is given, where $x_1 = Q$ is the quantity of the contract gas. We formulate the problem as a finite time horizon MDP:

$$V_{T+1}(x_{T+1}, p_{T+1}, D_{T+1}) = 0, \quad (x_{T+1}, p_{T+1}, D_{T+1}) \in X \times \mathbb{R}_+ \times \mathcal{D},$$

$$V_t(x_t, p_t, D_t) = \min_{a_t \in \mathcal{A}} p_t(D_t - a_t) + \mathbb{E}[V_{t+1}(x_{t+1}, p_{t+1}, D_{t+1}) | p_t, D_t], \quad (x_t, p_t, D_t) \in X \times \mathbb{R}_+ \times \mathcal{D}; t \in \mathcal{T}.$$  

$V_t(x_t, p_t, D_t)$ denotes the optimal value of the cost from period $t$ onward for a given state $(x_t, p_t, D_t)$ at stage $t \in \mathcal{T} \cup \{T + 1\}$. Stage $T + 1$ is introduced to facilitate the clarification of bound rule. We use $\tau$ to denote a random variable in the rest of this paper.

### 2.2 Optimal Base Stock Policy

This subsection is dedicated to deriving an optimal policy for dynamic allocation of contracted gas. To begin with, the power demands $D_t$, $t = 1, \ldots, T$ are assumed to be independently and identically distributed. For the ease of ensuing exposition, we then define $U_t(x, p)$ as follows:

$$U_t(x, p) = \mathbb{E}[V_{t+1}(x, p_{t+1}, D_{t+1}) | p_t = p, D_t], \quad t \in \mathcal{T}, (x, p) \in X \times \mathbb{R}_+.$$  

(2.3)

As the demand process is assumed to be independently distributed, taking expectation with respect to $D_{t+1}$ conditional on $D_t$ makes $U_t(x, p)$ independent of $D_t$. Subsequently, to guarantee that the optimal value of the minimization problem (2.2) is finite for any stage and state, it is necessary to introduce Assumption 1.

**Assumption 1.** It holds that $\mathbb{E}[\hat{p}_t | p_t = p] < \infty, j = 1, 2, \ldots, T - 1, k \in \mathcal{T}$ with $k \geq j$, and $p \in \mathbb{R}_+$.

Provided that Assumption 1 holds throughout the paper, we establish the convexity of the value function and show that a base stock policy is optimal in the following proposition.

**Proposition 2.1** (Optimal Base Stock Policy). In any stage $t \in \mathcal{T} \cup \{T + 1\}$, $V_t(x_t, p_t, D_t)$ is convex in $x_t$ for given $p_t$ and $D_t$. Also, there exists a base stock target $BS_t(p_t)$, which only depends on price $p_t$ in current stage $t$, such that the optimal contracted gas delivery amount for any given state $(x_t, p_t, D_t)$ is

$$a_t^*(x_t, p_t, D_t) = \begin{cases} \min\{C_d, D_t, (x_t - BS_t(p_t))\}, & \text{if } x_t \geq BS_t(p_t); \\ 0, & \text{if } x_t < BS_t(p_t). \end{cases}$$

(2.4)

**Proof:** The proof of the convexity of $V_t(x_t, p_t, D_t)$ in $x_t$ is similar to that of Proposition 1 in [8] and is therefore omitted in this paper. In order to show that the base stock policy is optimal, we focus on stage $t \in \mathcal{T}$. Pick any state $(x_t, p_t, D_t)$ in stage $t$. To determine the optimal action $a_t$ in this state, we relax the withdraw capacity constraint and demand restriction on $a_t$ and use $x_{t+1} = x_t - a_t$ as a new decision variable. Substituting $a_t$ with $x_t - x_{t+1}$ into (2.2), the optimization problem can be reformulated as:

$$\min_{x_{t+1} \in [0, x_t]} (D_t - x_t + x_{t+1}) p_t + U_t(x_{t+1}, p_t)$$

(2.5)

Consider a particular case with $x_t = Q$, then (2.5) is equivalent to

$$\min_{x_{t+1} \in [0, Q]} x_{t+1} p_t + U_t(x_{t+1}, p_t)$$

(2.6)

By expectation preserves convexity, the convexity of $V_{t+1}(x_{t+1}, p_{t+1}, D_{t+1})$ in $x_{t+1}$ implies that the objective function of the relaxed minimization problem (2.6) is also convex in $x_{t+1}$. Meanwhile, since the objective function is independent of $D_t$, the corresponding optimal solution, denoted by $BS_t(p_t)$, does not depend on the demand $D_t$.

Now consider the minimization problem (2.5) for $x_t \in [BS_t(p_t), Q]$. Since $BS_t(p_t) \in [0, x_t]$, the optimal solution is achieved at $x_{t+1}^* = BS_t(p_t)$. Therefore the pertinent part of the decision policy holds when the withdraw capacity constraint and demand restriction are imposed on $a_t$. On the other hand, if $x_t < BS_t(p_t)$, the convexity of the objective function of (2.5) implies that the value of the objective function monotonously decreases in $x_{t+1}$ in the interval $[0, x_t]$. Hence, the minimum value is achieved at $x_{t+1}^* = x_t$. In another word, the optimal action is purchasing from spot market to satisfy all of the demand in current stage. In conclusion, the claimed base stock policy holds. □
The base stock level $BS_t(p_t)$ can be interpreted as an anticipated remaining amount of contracted gas for next stage $t+1$, which only depends on spot price $p_t$ in current stage $t$. If the remaining contract level $x_t$ is higher than the target level $BS_t(p_t)$, it is optimal to withdraw the contracted gas and make the remaining contract level in next stage as close to $BS_t(p_t)$ as possible. In the counterpart case, since no injection is allowed, the remaining contracted gas itself is closest to the anticipated level $BS_t(p_t)$, hence the optimal action is satisfying all of the demand in current stage by purchasing from spot market.

3. Contract Valuation

Although the optimal base stock policy enables the GENCO to value the bilateral contract, it requires Monte Carlo simulation to generate spot price and demand sample paths in the computation, which discourages us to employ it directly. In this section, we investigate an alternative way to evaluate the value of the bilateral contract for both the GENCO and the natural gas supplier.

3.1 Contract Valuation for the GENCO

It is noteworthy that we can also evaluate the value of the contract for the GENCO directly from the MDP model (2.1)-(2.2). Recall that the GENCO has two options to satisfy its demand: either engage into a bilateral contract to lock a proportion of gas supply and procure the rest from spot trading, or purchase all of required natural gas from spot market without signing any contract. In a risk-neutral environment, the expected total procurement cost in these two scenarios are assumed to be equal, i.e. $V_1(Q, p_1, D_1) + J_m(Q) = V_1(0, p_1, D_1)$, where $J_m(Q)$ represents the value of the contract for the GENCO. Equivalently, for given $p_1$ and $D_1$, $J_m(Q)$ can be obtained by

$$J_m(Q) = \hat{V}_1(Q, p_1, D_1)$$

(3.1)

Where,

$$\hat{V}_{t+1}(x_{t+1}, p_{t+1}, D_{t+1}) = 0, \quad (x_{t+1}, p_{t+1}, D_{t+1}) \in X \times \mathbb{R}_+ \times \mathcal{D}$$

$$\hat{V}_t(x_t, p_t, D_t) = \max_{a_t \in \mathcal{R}} p_t a_t + \mathbb{E}[\hat{V}_{t+1}(x_{t+1} - a_t, p_{t+1}, D_{t+1})|p_t, D_t], \quad (x_t, p_t, D_t) \in X \times \mathbb{R}_+ \times \mathcal{D}; t \in T$$

(3.2)

Note that $J_m(Q)$ reflects the maximum price that the GENCO would like to pay for the $Q$ amount of bilateral contract. To establish a mutually acceptable contract price, we also need the minimum price $J_s(Q)$ that the gas supplier can offer for the same amount of contract.

3.2 Contract Valuation for the Gas Supplier

In this subsection, we evaluate the value of the bilateral contract for the natural gas supplier. The time horizon in this problem is the same as the one presented in Figure 2.1. At the beginning of the planning horizon, the gas supplier engages into a bilateral contract with a GENCO, stipulating $Q$ amount of natural gas to supply in the planning horizon. Most importantly, we assume that the supplier has the privilege to determine the contract delivery amount in each time period that the GENCO has to follow. The reason for this assumption is twofold: First, we are seeking for $J_s(Q)$—the minimum price that the gas supplier would like to offer, which should only depend on the supplier’s own operation strategy. In this way, the supplier can make profit as long as the compromised price is higher than $J_s(Q)$ for the $Q$ amount contract; Second, if the supplier is enforced to deliver the contracted gas required by the GENCO, they will share the same uncertainty information. As a consequence, $J_s(Q)$ will coincide with $J_m(Q)$. Therefore, to avoid triviality, the gas supplier’s action on contract delivery in each stage should be independent of the GENCO’s contracted gas allocation decision. To supply the stipulated contract, the gas supplier produce natural gas from raw material, for example, liquified natural gas. Besides, the gas supplier can also access to spot market to purchase natural gas or sell produced natural gas to make profit at any time.

At the beginning of period $t$, the gas supplier checks the remaining contract amount $x_t$ and observes the revealed gas spot price $p_t$. Note that the spot prices $p_t, t = 1, 2, \cdots, T$ observed by the supplier are the same as those revealed to the GENCO. Then the supplier makes joint decisions on the natural gas production amount $q_t$ and the contract delivery amount $a_t^s$. The production action $q_t$ and delivery action $a_t^s$ are commonly constrained by the production capacity $C_p$ and the delivery capacity $C_d$, both of which are positive constants. Besides, it is not necessary for the supplier to deliver natural gas which is beyond the stipulated supply amount, i.e. $a_t^s \leq x_t$. Hence the feasible action set for $q_t$ and $a_t^s$ is defined as $\mathcal{Q} \times \mathcal{A}_t = \{ (q_t, a_t^s) : 0 \leq q_t \leq C_p, 0 \leq a_t^s \leq \min(C_d, x_t) \}$. The production action $q_t$ yields an immediate production cost $l(q_t)$, which only depends on the production level $q_t$. When the contract delivery amount $a_t^s$ is determined, a recourse action is taken. If $a_t^s \geq q_t$, the gas supplier purchases $a_t^s - q_t$ amount from spot market, otherwise sells $q_t - a_t^s$ amount to the spot market. The recourse action directly brings about extra purchase cost (or
sale revenue \( p_t(a_t^i - q_t) \). Then the problem can be formulated as below:

\[
R_{t+1}(x_{T+1}, p_{T+1}) = \begin{cases} 
0, & \text{if } x_{T+1} = 0; \\
+\infty, & \text{if } x_{T+1} > 0.
\end{cases} \quad (3.3)
\]

\[
R_t(x_t, p_t) = \min_{(q_t, a_t^i) \in Q \times \mathbb{R}_+^i} l(q_t) + p_t(a_t^i - q_t) + \mathbb{E}[R_{t+1}(x_t - a_t^i, \hat{p}_{t+1}) \mid p_t], \quad (x_t, p_t) \in X \times \mathbb{R}_+; t \in T \quad (3.4)
\]

Where \( R_t(x_t, p_t) \) represents the optimal value function in period \( t \) for a given state \((x_t, p_t)\). The termination rule described in (3.3) is tailored to the requirement that the gas supplier has to supply all of the stipulated contact amount. Hence, an infinite penalty cost occurs if the contract is not completely supplied in the end of planning horizon.

Similar to the definition of \( J_m(Q) \), \( J_s(Q) = R_t(Q, p_t) - R_t(0, p_t) \). For any given initial spot price \( p_1 \), \( J_s(Q) \) can be simplified as:

\[
J_s(Q) = \hat{R}_t(Q, p_1) \quad (3.5)
\]

Where

\[
\hat{R}_{t+1}(x_{T+1}, p_{T+1}) = \begin{cases} 
0, & \text{if } x_{T+1} = 0; \\
+\infty, & \text{if } x_{T+1} > 0.
\end{cases} \quad (3.6)
\]

Remark: The formulations of \( J_m(Q) \) and \( J_s(Q) \) imply that \( J_m(Q) \) is equivalent to the maximal expected profit by reselling the contract back to the spot market, while \( J_s(Q) \) is equivalent to the minimum expected cost by purchasing from the spot market to supply the contract. The finding is consistent with conclusion in [14].

4. Contract Price Determination

With the knowledge of contract value-quantity curves \( J_s(Q) \) and \( J_m(Q) \), it is now possible for the contracting parties to negotiate and reach an agreement as long as there exists a \( Q \) such that \( J_s(Q) \leq J_m(Q) \).

The shadow region in Figure 4.1 presents the feasible set of the \((Q, J)\) pairs \( I = \{(Q, J) : 0 \leq Q \leq Q_{\text{max}}, J_s(Q) \leq J_m(Q) \} \). It can be shown that the feasible set \( I \) is always non-empty, within which the contracting parties can negotiate to achieve a mutually acceptable \((Q, J)\) combination.

This negotiation process can be formulated as an asymmetric Nash bargaining game. Consider a feasible \((Q, J) \in I \). \( J_m(Q) - J \) and \( J - J_s(Q) \) compute the utility that the GENCO and the gas supplier can gain from the bilateral contract, respectively. In light of the observation in reality that usually one of the contracting parties takes an advantageous position in the price negotiation process, we incorporate asymmetric bargain powers in our model. The bargain powers of the GENCO and the supplier are represented as \( \mu \) and \( 1 - \mu \), respectively. Where \( \mu \in [0, 1] \). The goal is to select the optimal \((Q, J)\) combination within the feasible region \( I \) such that the total utilities gained from bilateral contract are maximized:

\[
\max_{(Q, J) \in I} (J_m(Q) - J)^\mu (J - J_s(Q))^{1-\mu} \quad (4.1)
\]

The existence and uniqueness equilibrium of the Nash bargaining game are established in Proposition 4.1 as follows:

**Proposition 4.1.** There exists a unique equilibrium \((Q^*, J^*)\) for the Nash bargaining game, where

\[
Q^* = \arg\max_Q (J_m(Q) - J_s(Q)) \quad (4.2)
\]
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\[ J^* = \mu J_s(Q^*) + (1 - \mu) J_m(Q^*) \]  (4.3)

**Proof:** Define the objective function

\[ f(J, Q) = (J_m(Q) - J)^\mu (J - J_s(Q))^{1-\mu} \]  (4.4)

Then the partial derivative of \( f(J) \) on \( J \) is

\[ \frac{\partial f(J, Q)}{\partial J} = -\mu \theta^{1-\mu} + (1 - \mu) \theta^{-\mu} \]  (4.5)

where \( \theta = \frac{J - J_s(Q)}{J_m(Q) - J} \). By the first order condition, we obtain that

\[ (1 - \mu) (J_m(Q) - J^*) = \mu (J^* - J_s(Q)) \]  (4.6)

which is equivalent to

\[ J^*(Q) = \mu J_s(Q) + (1 - \mu) J_m(Q) \]  (4.7)

The second derivative of \( f(J, Q) \) on \( J \) is

\[ \frac{\partial^2 f(J, Q)}{\partial J^2} = -\mu(1 - \mu) (J_m(Q) - J_s(Q)) (J_m(Q) - J)^{-2+\mu} (J - J_s(Q))^{-1-\mu} \leq 0 \]  (4.8)

Hence \( J^*(Q) \) is the unique global optimal solution of the Nash bargaining model for a given \( Q \). Thus \( J^*(Q) \) can be plugged into the maximization problem (4.1), the problem becomes:

\[ \max_{(Q, J) \in l} (J_m(Q) - J)^{1-\mu} (J - J_s(Q))^{1-\mu} \max_{0 \leq Q \leq Q_{max}} (J_m(Q) - J_s(Q)) \]  (4.9)

Therefore, the optimal contract quantity is

\[ Q^* = \arg \max_Q (J_m(Q) - J_s(Q)) \]  (4.10)

And the corresponding optimal contract price is

\[ J^* = \mu J_s(Q^*) + (1 - \mu) J_m(Q^*) \]  (4.11)

\[ \Box \]

The Nash equilibrium solution provides several managerial insights on contract price and quantity determination. First, the optimal contract quantity only depends on the gap of the contract values between those two contracting parties, regardless of their bargaining powers. Hence the optimal contract quantity can be determined as long as both parties have a good knowledge of the contract value-quantity curves. Second, equation (4.6) implies that the utility that one gains from the bilateral contract is proportional to its bargain power. The higher the bargain power is, the more it can benefit from the bilateral contract. Third, optimal contract price is a weighted average of their contract valuations with bargain powers serving as weighting coefficients. It is ease of implementation for the contracting parties to determine the optimal contract quantity and price separately in sequence in the negotiation process, which significantly simplifies the negotiation strategy.

**5. Numerical Study**

In the section, we take a one month short term natural gas supply management problem as an illustrative example, which can be easily scaled up to a one year schedule problem. The initial gas contract amount is normalized to 10 for simplicity. To solve the MDP model by standard backward dynamic programming, it is necessary to discretize the available remaining contract amount \( x \). To be specific, \( x \) is restricted to 101 evenly spaced levels \((0, 0.1, \cdots, 10)\), so that the computation of optimal value function in each stage can be focused only on these 101 discrete levels. Withdraw capacity is set to 0.6, which is determined by the contract clause and physical pipeline transportation limit. Daily electricity demand \( D_t \), represented by the volume of natural gas required to produce the equivalent amount of electricity, is discretized as shown in Table 5.1.

<table>
<thead>
<tr>
<th>Demand</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.03</td>
<td>0.08</td>
<td>0.15</td>
<td>0.32</td>
<td>0.22</td>
<td>0.12</td>
<td>0.06</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 5.1: Discretized demand distribution

The natural gas spot price evolution process is described by a single-factor mean-reversion model [13] as follows:

\[ dX_t = -\kappa (X_t - \xi) dt + \sigma dW_t \]  (5.1)

Where \( X_t = \ln(p_t) \) and \( W_t \) denotes a wiener process. In the model, the mean reversion rate \( \kappa \), long term mean reversion level \( \xi \) and volatility \( \sigma \) are assumed to be positive constants. These parameters are estimated with 252 historical natural gas spot price data obtained from NYMEX ranging from Jan 2 to Dec 31 in 2009. The estimates of the parameter
values are displayed in Table 5.2. Moreover, to facilitate standard backward recursion, a trinomial scenario tree is built for the spot price evolution process, a simple adaption of method proposed in [13].

Table 5.2: Estimated parameter values for the mean reversion model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>κ</th>
<th>σ</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.0336</td>
<td>0.048</td>
<td>1.436</td>
</tr>
</tbody>
</table>

In the experiment, we set $\mu = 0.5$. Figure 5.1 plots the contract value-quantity curves $J_n(Q)$ and $J_s(Q)$ in terms of the GENCO and the gas supplier, respectively. When stipulated contract quantity is not very large (for example $Q \leq 11$), $J_n(Q)$ would increase and $J_s(Q)$ would decrease as the variation of spot price $\sigma$ goes up. It means that both parties can gain from the contract by managing its contract allocation strategically. However, when the stipulated contract volume is very large, $J_s(Q)$ also increases as spot price variation increases. Recall that the contract value $J_s(Q)$ in terms of the gas supplier is equivalent to the minimal expected cost by purchasing from the spot market to supply the contract. In the case of large stipulated contract quantity $Q$, the gas supplier has to purchase from the spot market to supply the contract even if the spot price is very high in certain periods, giving rise to the unexpected increase of $J_s(Q)$ for large $Q$ when spot price variation increases.

![Figure 5.1: The contract value-quantity curves for different spot price variations](image)

It is also noteworthy that for fixed spot price variation $\sigma$, the contract value curves $J_n(Q)$ and $J_s(Q)$ would cross each other somewhere. This phenomenon can be attributed to demand uncertainty in the decision making process. It is clear that $J_s(Q) \leq J_n(Q)$ holds for small stipulated contract volume $Q$. When the stipulated contract volume is large (for example $Q > 16$), however, it is probably that the contract gas would not be used up due to demand uncertainty. To some extents, the contract value $J_n(Q)$ is computed with respect to a portion of contract quantity rather than the whole contract volume, since the remaining amount in the end of planning horizon is supposed to be worthless and discarded. On the other hand, the gas supplier does not suffer from power demand uncertainty, and thus evaluates the value of the bilateral contract $J_s(Q)$ in terms of total stipulated contract quantity $Q$. Hence these two curves will intersect with each other somewhere.

To further discover the effect of spot price variation $\sigma$ on the optimal contract quantity $Q^*$ and price $J^*(Q^*)$, we compute the optimal contract quantity and price under different price variations $\sigma$ varying from 0.05 to 0.25 at the increment of 0.025. As can be seen from left panel of Figure 5.2, the optimal contract quantity $Q^*$ is first increasing and then decreasing as spot price variation $\sigma$ goes up. It is because when price variation $\sigma$ gradually increases, it is more likely to reveal a high or low spot price. In this case, both contracting parties are inclined to expand the contract volume since both of them can gain benefits from the contract. However, if the spot market becomes extremely volatile, $J_s(Q)$ would increase a lot due to the highly volatile spot price, especially for a large contract volume, as presented above. As a result, it might not be profitable for the supplier to expand the bilateral contract. Therefore the optimal quantity for bilateral contract might decline for extremely volatile spot market. The trend of optimal contract price change is similar to that of optimal contract quantity.
It is possible that higher optimal contract quantity might lead to increasing optimal contract price. Therefore, it is more appropriate to examine the optimal unit contract price, the ratio between the optimal contract price and corresponding optimal quantity. The right panel of Figure 5.2 presents the optimal unit contract price with respect to different spot price variations $\sigma$ and negotiation powers $\mu$. It is obvious that the optimal unit contract price is linearly decreasing in the negotiation power $\mu$, as implied by the Nash equilibrium proposition 4.1. When $\mu < 0.5$, the unit contract price is increasing in the spot market variation $\sigma$. This is because low $\mu$ means that the contract price is more influenced by $J_m(Q)$ according to equation (4.7). On the contrary, if $\mu > 0.5$, the GENCO takes the advantageous position in the bargaining process. It prefers to squeeze the contract price as low as possible. Combining with the analysis of the effect of spot price variation on $J_m(Q)$ and $J_s(Q)$, we can conclude that the optimal unit contract price increases in spot price variation $\sigma$ for a low $\mu$ value and decreases in spot price variation $\sigma$ for a high $\mu$ value.

6. Conclusion
In this paper, we investigate a multistage stochastic natural gas supply problem for a GENCO to aid contract price and quantity determination in deregulated environment. A novel scheme is developed to determine the optimal contract price and the optimal gas amount to engage in a bilateral contract. We first establish an optimal base stock policy for dynamic allocation of contracted gas in the face of volatile spot price and stochastic demand, which is ease of implementation in practice. Subsequently, we evaluate the value of the bilateral contract for the GENCO and the supplier, respectively. Moreover, by incorporating bargaining powers, the contract negotiation process is modeled as an asymmetric Nash bargaining game. The unique equilibrium solution implies that the optimal contract quantity is independent of the bargaining powers and the optimal contract price is a weighted average of the contract valuations. In the end, the feasibility of the model is validated by computational results under various market conditions. Numerical study also demonstrates that the optimal contract quantity (or price) is nonmonotonic in spot price variation.

Several directions for future extension are discussed as below. In a typical natural gas market, the GENCO would compete with other buyers and negotiate with multiple gas suppliers. It is straightforward to extend the framework to a more general situation with multiple participants. Under such circumstance, it is necessary to adopt cooperative or non-cooperative game theory to analyze the complicated contract negotiation process. Another challenging extension is to consider multiple time scale energy portfolio optimization problem, where a coarse monthly financial trading in the forward market and a fine daily operation scheduling are integrated together. Introducing the forward market inevitably enlarges the scale of the problem a lot and probably makes the problem computationally intractable. Hence, it might entail some approximation techniques, such as approximate linear programming and approximated dynamic programming, to solve the problem.

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References


