Determining the Retailer’s Replenishment Policy Considering Multiple Capacitated Suppliers and Price-sensitive Demand

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Abstract

A mixed integer nonlinear programming model is presented to find the optimal replenishment policy for a particular type of product for the case of a single retailer and multiple potential suppliers. Each supplier offers all-unit quantity discounts as an incentive mechanism. Multiple orders are allowed to be submitted to the selected suppliers during a repeating order cycle. The demand rate is considered to be not constant but dependent upon the selling price. The model provides the optimal number of orders and corresponding order quantities for the selected suppliers, and the optimal demand rate and selling price that maximize the total profit per time unit under suppliers’ capacity and quality constraints. A numerical example is presented to illustrate the results of the proposed model.

Key words: Supplier selection, Price-sensitive demand, Supply chain inventory, All-unit quantity discounts, Mixed integer nonlinear programming model.

1. Introduction

Supply chain inventory management faces the challenge of a price-sensitive nature of demand; it is very common in practice that products’ demand varies with the selling price as it plays a significant role in attracting consumers. Therefore, developing supply chain inventory models involving joint pricing and ordering/production decisions have gained great attention in the literature and practice. In 1955, Whitin [15] was the first to incorporate the concept of linking inventory theory (i.e. the classic Economic Order Quantity EOQ) and economic price theory in which he considered the demand rate to be linearly dependent on the product’s price. Then, Kunreuther and Richard [9] studied the interrelationship between pricing and inventory decisions, and determined the retailer’s optimal pricing and ordering decisions.

Scholars have focused the research on studying pricing and ordering decisions with considering price sensitive demand and quantity discounts. Abad [1] found the optimal selling price and lot size when the supplier offers all-unit quantity discounts considering two types of price varying demands, namely, linear and negative power function of price, and then in [2] he considered the problem when the supplier offers incremental quantity discounts. Thereafter, Burewell et al. [6] and Yildirmaz et al. [16] considered the same problem proposed by Abad [1] but with the incorporation of transportation costs to determine optimal pricing and lot-sizing decisions. Wang and Wang [14] determined the supplier’s optimal quantity discount policy for a set of independent and heterogeneous buyers under price sensitive demand. Recently, Lin [10] proposed an integrated inventory model to find the optimal pricing and ordering strategies when quantity discounts and price sensitive demand are considered.

The aforementioned studies assumed that the suppliers have sufficient capacity and perfect products’ quality. However, in practice suppliers have some limitations on capacity, quality, delivery, price, etc. Accordingly, the problem is viewed as a supplier selection problem that is defined as a multi-criteria problem that aims at selecting a group of suppliers among a set of potential suppliers based on retailer’s qualitative and quantitative criteria. Some recent comprehensive surveys in reviewing pervious work about supplier selection problem are given in Aissaoui [5], Ho et al. [8], and Agarwal el al. [4].

Research on developing inventory models that simultaneously handle pricing and ordering decisions with considering capacity limitations include Deng and Yano [7] who considered the case of a capacitated manufacturer faces a price sensitive demand for which they studied the optimal prices and production quantities for a constant and
time-varying capacity. Smith [13] jointly determined the optimal pricing and product master planning over a discrete time multi-period horizon with considering capacity and inventory constraints. Recently, Razaei [12] proposed a multi-objective nonlinear programming model to find the optimal selling price and lot-size of multiple products in multiple periods considering budget, storage, and supplier capacity limitations.

In this paper, we consider a supply chain inventory problem for a particular type of product with multiple suppliers. Each supplier offers all-unit quantity discount to motivate the retailer to place lager order for a lower unit cost. The retailer faces a price-sensitive demand which is in particular a negative power function of the selling price. In addition, the retailer is not restricted to place at most one order per an order cycle. Thus, the goal is to propose a supplier selection model that simultaneously finds the retailer’s optimal selling price, number of orders and the corresponding order quantities for the selected suppliers that maximize the retailer’s total profit per time unit under suppliers’ limitations on capacity and quality. The remainder of this paper is organized as follows. Section 2 presents our notation, problem description, and model development. Section 3 contains a computational study for the proposed model. Section 4 concludes with a brief summary and presents suggestions for future research.

2. Problem description and model development

Let \( \{i = 1, ..., n\} \) denote the set of \( n \) potential suppliers who deliver a particular type of product and \( c_i \) denote suppliers’ capacity (production) rate. Also, let \( q_i \) denote suppliers’ quality level which represents the percentage of acceptable units and \( k_i \) denote the setup cost for supplier \( i \). The retailer’s unit purchase price is defined by the following all-unit quantity discounts mechanism offered by supplier \( i \):

\[
v(Q_i) = \begin{cases} 
0 & \text{if } Q_i = 0 \\
v_{i1} & \text{if } 0 < Q_i < u_{i1} \\
v_{i2} & \text{if } u_{i1} \leq Q_i < u_{i2} \\
\vdots & \text{if } u_{i(a_i-2)} \leq Q_i < u_{i(a_i-1)} \\
v_{i(a_i-1)} & \text{if } u_{i(a_i-2)} \leq Q_i < u_{i(a_i-1)} \\
v_{i(a_i)} & \text{if } u_{i(a_i-1)} \leq Q_i < \infty 
\end{cases}, \quad i = 1, 2, \ldots, n,
\]

where \( Q_i \) is the purchased lot size from supplier \( i \), and \( u_{i0} < u_{i1} < \cdots < u_{i(a_i-1)} < u_{i(a_i)} = \infty \) are the sequence of quantities at which the unit price changes, and \( a_i \) is the number of quantity discount intervals offered by supplier \( i \). For instance, the purchasing cost for a lot size \( Q_i \) is \( v_{ij}Q_i \), if \( u_{ij} \leq Q_i < u_{ij} \), where \( u_{ij} \) is supplier \( i \)'s strict upper bound of discount interval \( j \) and \( v_{ij} \) is the unit price, \( j = 1, 2, \ldots, a_i \), and \( v_{i(a_i)} > \cdots > v_{i(a_i-2)} > v_{i(a_i-1)} > 0 \). Let \( Y_{ij} \) denote a binary variable that equals to one if discount interval \( j \) is selected for supplier \( i \), and equals zero otherwise.

The retailer’s demand rate is a decreasing function of the selling price \( P \) given by a constant price elasticity function \( D = \propto P^{-e} \), where \( \propto \) and \( e \) are the scaling factor, and the price elasticity index, respectively. We consider the contribution made by Mendoza and Ventura [11] who recommend to allow multiple orders to be submitted to the selected suppliers during an order cycle (i.e. \( J_{ij} \) orders are allowed to be placed to supplier \( i \) in interval \( j \)). In addition, let \( r \) represent the inventory holding cost rate and \( q_{\min} \) denote the minimum acceptable quality level for the retailer. Thus, the goal is to find the optimal selling price \( P \), number of orders \( J_{ij} \) and the corresponding order quantity \( Q_i \) for the selected supplier that maximize the retailer’s total profit per time unit subject to quality and capacity constraints.

We first introduce the repeating order cycle time \( T_C \). Recall that in our model, multiple orders can be allocated to the selected suppliers within an order cycle. Now, if the retailer receives an order quantity of size \( Q_i \) from supplier \( i \), then \( T_i \) is the time to consume that order under price-elastic demand, which equals to

\[
\frac{Q_i}{D} = \frac{Q_i}{\propto P^{-e}}.
\]
Hence, the total time to consume all the units ordered from supplier $i$ is equal to $T_i \sum_{j=1}^{a_i} J_{ij} = (Q_i / \alpha P^{-e}) \sum_{j=1}^{a_i} J_{ij}$. Accordingly, the total repeating order cycle time $T_c = \sum_{i=1}^{n} T_i \sum_{j=1}^{a_i} J_{ij} = \sum_{i=1}^{n} (Q_i / \alpha P^{-e}) \sum_{j=1}^{a_i} J_{ij} = Q / \alpha P^{-e}$, where $Q$ denotes the total order quantities received from all supplier within an order cycle (i.e. $\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}$).

Note that, the number of orders submitted to each supplier and the corresponding order quantities will be repeated in each cycle. Therefore, the mixed integer nonlinear programming model (M1) is as follows [3]:

$$
\text{Max. } Z = \alpha P^{1-e} - \frac{1}{Q} \left[ \alpha P^{-e} \sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij} + (1/2)r \sum_{i=1}^{n} Q_i^2 \sum_{j=1}^{a_i} J_{ij} v_{ij} + \alpha P^{-e} \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij} v_{ij} \right],
$$

subject to

$$
Q = \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}, \quad (1)
$$

$$
\alpha P^{-e} Q_i \sum_{j=1}^{a_i} J_{ij} \leq c_i, \quad i = 1, ..., n, \quad (2)
$$

$$
\frac{\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}}{\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}} \geq q_a \Rightarrow \sum_{i=1}^{n} Q_i(q_i - q_a) \sum_{j=1}^{a_i} J_{ij} \geq 0, \quad (3)
$$

$$
Q_i \leq \sum_{j=1}^{a_i} u_{ij} Y_{ij}, \quad i = 1, ..., n \& j = 1, ..., a_i, \quad (4)
$$

$$
Q_i \geq \sum_{j=1}^{a_i} u_{ij-1} Y_{ij}, \quad i = 1, ..., n \& j = 1, ..., a_i, \quad (5)
$$

$$
\sum_{j=1}^{a_i} Y_{ij} \leq 1, \quad i = 1, ..., n, \quad (6)
$$

$$
\sum_{i=1}^{n} \sum_{j=1}^{a_i} J_{ij} \leq m, \quad (7)
$$

$$
J_{ij} \leq m Y_{ij}, \quad i = 1, ..., n \& j = 1, ..., a_i, \quad (8)
$$

$$
J_{ij} \geq 0, \text{ integer }, \quad i = 1, ..., n \& j = 1, ..., a_i, \quad (9)
$$

$$
Q_i \geq 0, \quad i = 1, ..., n, \quad (10)
$$

$$
P \geq 0, \quad (11)
$$

$$
Y_{ij} \in (0,1), \quad i = 1, ..., n \& j = 1, ..., a_i. \quad (12)
$$

The goal of the objective function is to maximize the retailer’s total profit per time unit which equals to the total sales revenue per time unit $PD$ minus the total replenishment and inventory costs per time unit which consists of setup cost, holding cost, and purchasing cost. Model (M1) is subject to a number of constraints; constraint (1) represents the total order quantities received from all suppliers within an order cycle. Constraint (2) guarantees the proportional demand per time unit that a certain supplier achieves is less than or equal to the corresponding capacity of that supplier. Constraint (3) shows that the average quality level obtained from all suppliers has to be greater than or equal to the retailer’s minimum acceptable quality level. Constraints (4) and (5) make sure that the ordered quantity is within the quantity discount interval, and constraint (6) guarantees that at most one of the supplier’s quantity discount intervals is selected. Constraint (7) represents the maximum number of orders that can be placed to the selected suppliers in a repeating order cycle. Constraints (8) and (9) make sure that the number of orders allocated to a certain supplier is integer and less than or equal to $m$. Finally, constraints (10), (11) and (12) represent the non-negativity and binary conditions.
3. Numerical Example

In this section we consider the case of a single retailer and multiple potential suppliers. Table 1 shows all the related data for three potential suppliers. The retailer’s monthly demand rate is a decreasing function of the selling price: \( D = \alpha P^{-e} \), where \( \alpha = 3375000 \) and \( e = 3 \). The retailer’s inventory holding cost rate is 0.3 per month, and the retailer’s minimum acceptable quality level is 0.95. The goal is to determine the number of orders that need to be placed to the selected suppliers, the corresponding order quantities, and the optimal selling price.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>( j )</th>
<th>Lower Bound ( u_{j-1} )</th>
<th>Upper Bound ( u_j )</th>
<th>Unit Price ( P ) ($)</th>
<th>Quality Level ( q_j )</th>
<th>Capacity ( C ) (units/month)</th>
<th>Ordering Cost ( O ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>1</td>
<td>0</td>
<td>50</td>
<td>9</td>
<td>0.92</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>50</td>
<td>100</td>
<td>8.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>100</td>
<td>150</td>
<td>8.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>150</td>
<td>200</td>
<td>8.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>200</td>
<td>( \infty )</td>
<td>8.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supplier 2</td>
<td>1</td>
<td>0</td>
<td>75</td>
<td>9.8</td>
<td>0.95</td>
<td>350</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>75</td>
<td>150</td>
<td>9.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>150</td>
<td>225</td>
<td>9.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>225</td>
<td>( \infty )</td>
<td>9.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supplier 3</td>
<td>1</td>
<td>0</td>
<td>100</td>
<td>10.5</td>
<td>0.98</td>
<td>250</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>200</td>
<td>10.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>200</td>
<td>( \infty )</td>
<td>10.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This problem was formulated and solved using LINGO 13.0 with global optimizer on a PC with INTEL(R) Core (TM) 2 Duo Processor at 2.10 GHz and 4.0 GB RAM. Now, to determine the optimal value of \( m \) where the absolute maximum profit is obtained, we set \( m \) to a very large number and it has been found that at \( m = 29 \) the absolute maximum profit is obtained \( $4180.98/month \). \( Q_{15} = 7 \), \( Q_{24} = 14 \), \( Q_{33} = 8 \), \( Q_{15} = 567.67 \) units, \( Q_{24} = 397.37 \) units, \( Q_{33} = 496.71 \) units, \( P = $15.84 \), which result in a cycle time of 15.89 months. Notice that in Figure 1, as \( m \) increases, the order cycle time increases. Therefore, decision makers should select a reasonable value for \( m \) that achieves a high average monthly profit and reaches a reasonably small order cycle time. Thus, constraint (7) is changed to equality and different values of \( m \) are considered. Figure 1 depicts the change in average monthly profit for \( m = 1, \ldots, 20 \). In this example, \( m = 4 \) ($4178.42/month) should be selected and it would result in a small decrease in profit of $0.06%/month comparing to the absolute maximum profit which can be justified by the advantages of reducing the cycle time to 2.17 months. Furthermore, in Figure 1, Label 1 denotes model \( M1' \) that restricts the retailer to place at most one order to the selected suppliers in an order cycle. As shown in Figure 1, at \( m = 3 \) the proposed model \( M1 \) and model \( M1' \) have the same profit value. Accordingly, it can be concluded that, restricting the retailer to place at most one order to the selected suppliers will obtain a suboptimal solution.

Figure 1. Model \( M1' \)'s behavior over different values of \( m \). [3]


4. Conclusions and Future Works

In this paper, we have proposed a mixed integer nonlinear programming model for a supplier selection problem in which the goal is to maximize the total profit per time unit under suppliers’ limitations on capacity and quality. In addition, to ensure a more realistic and practical situation, the suppliers in our model are adopting all-unit quantity discounts as an incentive mechanism, and the retailer’s demand is assumed to be price sensitive. Furthermore, multiple orders are allowed to be submitted to the selected suppliers during a repeating order cycle. In order to limit the length of the repeating cycle time to a reasonable value, we consider a predetermined value for the number of orders per order cycle. The proposed model simultaneously finds the optimal selling price, number of orders and the corresponding order quantities for the selected suppliers that maximize the retailer’s total profit per time unit. In addition, a numerical example is provided to illustrate the proposed model. Also, we show that a suboptimal solution is obtained when restricting the retailer to place at most one order to the selected suppliers per a repeating order cycle. Future research topics in this area include developing an integrated model that maximizes the retailer’s and suppliers’ profit. Also, our model can be extended by considering other conditions such as multiple products and multiple buyers.

References