Optimizing Costs and Emissions Due to Inventory Replenishment of Perishable Products

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Abstract

This paper presents a bi-objective, mixed integer programming model to manage inventory replenishment decisions for fixed-shelf life perishable products. These are products such as dairy, canned products, or pharmaceuticals, which have an expiration date. The model minimizes costs and CO₂ emissions due to inventory replenishment. The model presented in this paper is an extension of the economic lot-sizing model and captures the trade-offs between transportation and inventory costs, transportation mode and remaining shelf life of a product, and replenishment costs and emissions. The model considers different transportation modes; each mode has its own capacity, cost structure and lead time. Transportation modes considered include full truckloads, refrigerated trucks, and less-than-truckload. The model assumes a planning horizon of length T that repeats cyclically over time. The model is solved using a modified weighted sum approach. We perform extensive numerical experiments; the experimental results indicate that replenishment costs for perishable products are higher than for non-perishable products. Experimental results also indicate that replenishment costs for slow moving perishable products are higher than replenishment costs for fast moving perishable products.

Keywords
Bi-objective optimization, perishable products, economic lot size, carbon emission, multi-mode replenishment of inventories

1. Introduction

This paper presents an extension of the economic lot sizing (ELS) problem for perishable products. The classical lot sizing problem determines a replenishment schedule that satisfies a given set of demands at minimum total cost. Problem inputs include the unit production cost, the unit inventory holding cost, and the set-up cost in each time period over a finite and discrete-time horizon. The mathematical model proposed focuses on the ELS problem for perishable products which have a fixed shelf life, such as dairy, canned products, or pharmaceuticals. We consider a planning horizon of length T that repeats cyclically over time. The objective is to minimize the total costs and greenhouse gas (GHG) emissions while replenishing perishable inventories. The paper focuses on GHG emissions due to transportation and inventory of perishable products.

The work presented in this paper is motivated by the increasing concerns about the impact that GHG emissions have on the quality of air and consequently, the quality of our lives. Transportation and other supply chain related activities are major contributors to GHG emissions [1]. The International Energy Agency (IEA) [2] states that 19% of the energy consumption and almost a quarter of the energy related CO₂ emissions worldwide result from transportation. In the USA, transportation comprises 28% of the total energy consumption [3]. The US
Environmental Protection Agency (EPA) estimates that during the period from 1990 to 2010, transportation-related emissions rose by 18% [4]. This is mainly due to the increased demand for travel, and the US vehicle fleet’s relatively stagnant fuel efficiency. This paper presents an extension of the ELS model which can be used within MRP systems in order to account for transportation and inventory replenishment decisions. The goal of this model is to minimize costs and GHG emissions resulting from inventory replenishment.

Inventory replenishment decisions for perishable products are more challenging as compared to non-perishable products for several reasons. First, transportation and order costs for products that have short shelf life are typically high. This is because the inventories are typically replenished frequently, and therefore, the average order size is small. For example, let us compare replenishment decisions for a product with a shelf life of one day with a product with a shelf life of one week. The inventories of the first product should be replenished daily at a minimum. Consequently, the replenishment quantity would be, at most, as high as daily demand. Within one week, at least seven shipments/orders are initiated. The inventory of the second product could be replenished just once a week, or less frequently. Second, transportation costs are high because companies would typically ship perishable products using transportation modes that have short lead times. This strategy results in a longer shelf life. Transportation modes that have short lead time are typically more expensive. Third, transportation and inventory costs are higher because most perishable products (such as milk) require refrigerating during transportation and inventory. Finally, in order to reduce lead time, companies may decide to purchase perishable products from local suppliers. While focusing on local suppliers decreases transportation costs, this may increase purchasing costs. Focusing on local suppliers decreases the supplier pool size by excluding suppliers located further away. This, consequently, decreases the likelihood that the facility will be able to identify suppliers (wholesalers) that can provide products at a competitive price.

Perishable products with fixed shelf life, such as canned food, do not need refrigerating. Other products, such as milk, would lose quality and value if not refrigerated. For these products, refrigeration or some other preservation technique is employed. Therefore, companies need to use refrigerated trucks and trains for transportation, as well as refrigerators for storage. These practices increase energy consumption and consequently GHG emissions from transportation and inventory operations.

The model we propose is a bi-objective, mixed-integer linear programming model which minimizes costs and environmental impacts due to replenishment decision for perishable products that have a fixed shelf life. The model we propose captures a number of trade-offs that facilities are faced with when replenishing the inventories for these products, such as: trade-offs between transportation costs and inventory holding costs, transportation costs and GHG emissions, transportation mode selection and remaining shelf life of a product.

This paper makes the following contributions to the existing literature. First, the model we propose can be used as a sub-module in MRP systems to help environmentally conscious companies with requirements planning when making replenishment decisions for perishable products. The model enables companies to determine (a) whether they should rely on local suppliers, or use suppliers located further away in order to replenish inventories of perishable products; (b) what mode of transportation should be used to optimize system performance; and (c) how frequently to replenish inventories. Second, the computational analysis provides very good insights about the relationship between costs and emissions, transportation and inventory costs, and transportation mode and inventory holding costs for perishable products. Finally, while there are papers in the literature that investigate ELS models for perishable products, the literature on ELS models that captures costs and GHG emissions due to inventory replenishment decisions is scarce. Additionally, to our knowledge, there is no literature on ELS models that minimize costs and emissions for perishable products with fixed shelf life.

2. Literature Review

The model presented here is an extension of classical Economic Lot Sizing model introduced by Wagner and Whitin [5]. The goal of the classical Economic Lot-Sizing (ELS) problem is to find a minimum-cost procurement plan for a single item with deterministic, non-stationary demand over a finite time-horizon in discrete-time. Most variants of the ELS problem implicitly assume that the items do not deteriorate between procurement and demand satisfaction. However, in some settings this assumption is unrealistic; consider, for example, agricultural and dairy products, fashion items, and items subject to rapid technological change. Nahmias [6], Raafat [7] and Li et al. [8] present extensive reviews of inventory replenishment models for perishable products. Nahmias provides an extensive review of ELS models for perishable items. Raafat reviews the literature on ELS models for continuously deteriorating items. Most recently, the work of Pierskalla [8] presents models for replenishment decisions within the supply chain
for blood, which is perishable and a vital product. Li et al. also provide an extensive review of some 100 recent papers on inventory management for perishable products. Eksioglu and Jin [10] present a model which captures the impact that the availability of multiple transportation modes for replenishing perishable inventories with fixed shelf life has on total inventory replenishment costs. Some other extensions of ELS problems with perishable products can be found in [11, 12, 13, 14].

The existing research on a product’s carbon footprint can be divided into those models which propose methods to measure and quantify carbon emissions in the supply chain due to processes such as transportation and inventory [15], and those which minimize the carbon footprint of a supply chain through changes in supply chain design and operations [16, 17]. Our study falls in the former stream of research which optimizes GHG emissions in the supply chain through inventory and transportation related decisions.

This work also is related to the literature on multiple objective optimization [18]. Wang et al. [19] present a bi-optimization model to design a supply chain network which optimizes costs and emissions. This paper extends the classical facility location model to consider transportation-related costs and emissions when making location decisions. Neto et al. [20] propose an algorithm to solve a multi-objective optimization model with three objectives: minimize costs, minimize cumulative energy demand and minimize waste in a reverse logistics network.

The work presented in this paper uses a bi-objective optimization model to minimize costs and emissions due to inventory replenishment decisions for perishable products with fixed shelf life. We assume multiple transportation modes and multiple suppliers are available in order to replenish the inventories. To the best of our knowledge, no other paper in the literature studies this problem.

3. Problem formulation

Consider the problem when a facility may use multiple suppliers to replenish inventory for a single perishable product. Some of the suppliers are local and therefore have a short lead time, while others are located further away. Each supplier may use different transportation modes to deliver the product. For example, full-truckloads can be used when the order quantity is high. Otherwise, less-than-truckload shipments are employed. Some suppliers located further away may use refrigerated trucks. As a result, the structure of the transportation cost differs by supplier and transportation mode. We assume that there are no capacity limits on transportation quantity, storage area or on inventory replenishment quantities.

\[ \sum_{t=1}^{3} b_{t} \]

\( b_{t} \) is the demand in period \( t \).

Let us assume that a planning horizon of length \( T \) is a typical one and repeats itself over time. All problem data are assumed cyclic with cycle length equal to \( T \) (\( b_{t+1} = b_{1}; b_{t+2} = b_{2}; \ldots \)), where \( b_{t} \) is the demand in period \( t \). As a
result, the inventory pattern at the facilities will be cyclic as well. The decisions in each time period include which supplier to select, which mode of transportation to use, and how much to order each time period. The following is a list of problem parameters and decision variables. Figure 1 provides a network representation of a problem with 1 supplier who has the option to select from 2 transportation modes to replenish inventories of a perishable product that has a shelf life of one period.

**Problem Parameters**

- \( M \): set of all the transportation modes available
- \( S \): set of all available suppliers
- \( s_{int} \): fixed set-up cost for replenishment mode \( m \) used by supplier \( i \) in period \( t \)
- \( h_t \): unit inventory cost in period \( t \)
- \( p_{int} \): unit procurement cost for mode \( m \) used by supplier \( i \) in period \( t \)
- \( b_t \): demand in period \( t \)
- \( A_m \): fixed cost of a replenishment mode \( m \)
- \( \hat{A} \): GHG emissions due to loading and unloading of one cargo container mode \( m \) used by supplier \( i \) in period \( t \)
- \( \hat{c}_{int} \): unit GHG emission due to transportation for mode \( m \) used by supplier \( i \) in period \( t \)
- \( \hat{h} \): unit GHG emissions due to holding one unit of inventory in period \( t \)
- \( w_m \): capacity of a cargo container for mode \( m \)
- \( k \): lifetime of the perishable item

**Decision variables**

- \( q_{int} \): the amount received from supplier \( i \) by mode \( m \) in period \( t \) to satisfy demand in period \( \tau \)
- \( y_{int} \): binary variable equal to 1 when replenishment mode \( m \) is used from supplier \( i \) in period \( t \), and 0 otherwise
- \( z_{int} \): number of cargo containers received from supplier \( i \) by mode \( m \) in period \( t \)

The following equation represents the total replenishment costs.

\[
TC(q, y, z) = \sum_{t=1}^{T} \sum_{i=1}^{S} \sum_{m=1}^{M} \sum_{k} (p_{int}q_{int\{t\}} + h_t q_{int\{t+1\}}) + s_{int}y_{int} + A_m z_{int}
\]  

(1)

The following equation represents the total emissions due to storage and transportation. In this study we consider only CO\(_2\) emissions, since they count for about 90\% of the total GHG emissions.

\[
TE(q, z) = \sum_{t=1}^{T} \sum_{i=1}^{S} \sum_{m=1}^{M} \sum_{k} c_{int} q_{int\{t\}} + h_t q_{int\{t+1\}} + \hat{A} z_{int}
\]  

(2)

The following is a mixed integer linear programming (MILP) formulation for this inventory replenishment problem.

\[
\min_{q, y, z} (TC(q, y, z), TE(q, z))
\]  

(3)

\[
\sum_{t=1}^{T} \sum_{i=1}^{S} \sum_{m=1}^{M} q_{int\{t\}} = b_t \quad \tau = 1, ..., T
\]  

(4)

\[
\sum_{i} q_{int\{t\}} \leq \sum_{i} h_t y_{int} \quad i = 1, ..., S; t = 1, ..., T; m = 1, ..., M
\]  

(5)

\[
\sum_{i} q_{int\{t\}} \leq W_{int} z_{int} \quad i = 1, ..., S; t = 1, ..., T; m = 1, ..., M
\]  

(6)

\[
q_{int\{t\}} \geq 0 \quad i = 1, ..., S; t = 1, ..., T; m = 1, ..., M; \tau = t, ..., T
\]  

(7)
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\[ y_{imt} \in \{0,1\} \quad i = 1,\ldots, S; t = 1,\ldots, T; m = 1,\ldots M \]  
\[ z_{imt} \in Z^+ \quad i = 1,\ldots, S; t = 1,\ldots, T; m = 1,\ldots M \]  

For notational convenience, in this formulation we have used the notation \([t] = (t+1) \mod{T+1}\) i.e., \(h_{t,T} = b_{t,1}\) for \(t=2,\ldots,T\). This objective function minimizes costs and emissions due to inventory replenishment decisions. The cost function includes production, setup, inventory holding, and transportation costs. The emission objective includes transportation, loading/unloading and storage related emissions.

Constraint set (4) ensures that demand in period \(\tau\) \((\tau = 1,\ldots,T)\) is satisfied. Constraint set (5) indicates that if a shipment is initiated from supplier \(i\) in period \(t\), then the amount shipped could be as great as the total demand in the following \(k\) periods. Recall that \(k\) represents the remaining shelf life of the product. Constraint set (6) identifies the number of cargo containers required to replenish inventories from supplier \(i\) in period \(t\), while (7) provide the non-negativity constraints, and (8) and (9) are impose integer constraints.

4. Solution approach
This section provides insights about the approach we use in order to solve the bi-objective optimization problem presented above. The mathematical model presented above is solved using GAMS/CPLEX solver on a personal computer with a 1.6 GHz Intel Core i7 processor and 6 GB RAM. The weighted sum method is one method which has been used frequently in the literature for multi-objective problems. This method transforms the objective function into a single objective by multiplying each objective with some pre-defined multipliers \(\lambda_1, \lambda_2 > 0\) such that \(\lambda_1 + \lambda_2 = 1\). The single objective problem is then solved for different values of these multipliers. The corresponding solutions are used to construct the Pareto frontier [21]. The following is the objective function of the single-objective problem.

\[
\text{Minimize} : \lambda_1 \sum_{t=1}^{T} \sum_{i=1}^{S} \sum_{m=1}^{M} \left( \sum_{\tau=t}^{t+k} (p_{imt}q_{imt+\tau}) + h_{t} q_{imt+\tau} \right) + s_{imt} y_{imt} + A_{mint} z_{imt} + \lambda_2 \sum_{t=1}^{T} \sum_{i=1}^{S} \sum_{m=1}^{M} \left( \sum_{\tau=t}^{t+k} c_{imt} q_{imt+\tau} + h_{t} q_{imt+\tau} + A_{mint} z_{imt} \right)
\]

We use a slightly different implementation of the weighted sum method. We set the value of \(\lambda_1 = 1\) and change the value of \(\lambda_2\) in each experiment. As a result, the relative value of the multipliers changes with each experiment. One can think of the values of \(\lambda_2\) as the cost of per unit of CO2 emissions. Additionally, one can consider \(\lambda_2\) to be the unit tax a facility would pay per unit of emission under a carbon tax mechanism. In this case; the objective function calculates the total costs due to replenishment and emissions in the supply chain.

5. Experimental Results
In this section we provide an example we order to demonstrate what we can learn from using the model described above. We also summarize results from the numerical experiments. The example considers the decisions faced by a retailer who can use 3 different suppliers in order to replenish its inventories for a perishable product. We assume that one unit of this product is equal to 1 ton, and the product’s shelf life is equal to \(k\) time periods. Supplier 1 is a local supplier who uses a less-than-truckload service provider for delivery. The delivery lead time for shipments from this supplier is zero. Supplier 2 is also a local supplier and has zero transportation lead time. However, supplier 2 is a wholesaler who provides discounted prices when the product is purchased in large quantities. This supplier sends full-truckload shipments using dedicated trucks. Supplier 3 is also a wholesaler who uses dedicated, refrigerated trucks for delivery. Due to the longer travel distance from this supplier, the corresponding delivery time is 2 days, although the unit replenishment cost for this supplier is the smallest. The order (set-up) and processing costs are the same for each supplier. Cargo container costs, which represent loading and unloading costs, are zero for the less-than-truckload service provider since she simply charges a fixed dollar amount per unit of product shipped. The dedicated trucks have a fixed capacity of 25 tons. Unit emissions are higher for shipments that use refrigerated trucks. We consider a time horizon of \(T = 10\) days, and a time period equal to 1 day. We assume that inventory holding costs equal $1/(ton*day) and inventory holding emissions are 0.5 kg/(ton*day). Table 1 summarizes the input data.
We test the performance of this facility under daily demands which vary from low levels \((b_i \sim N(3, 1.5) \text{ tons})\), to medium \((b_i \sim N(8, 4) \text{ tons})\), and high demand levels \((b_i \sim N(15, 7.5) \text{ tons})\). Note that the standard deviation for each distribution is such that the ratio of mean and standard deviation is the same. By ensuring this, we maintain the same relative demand variability for each demand level. Therefore, the results from our experiments will be impacted by the demand magnitude, but not relative demand variability.

Table 1: Problem Parameters

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Replenishment mode</th>
<th>Replenishment unit cost ((p))</th>
<th>Fixed order cost ((s))</th>
<th>Fixed cargo Cost ((A))</th>
<th>Capacity of mode ((W))</th>
<th>Fixed emissions ((\bar{A}))</th>
<th>Variable emissions ((\bar{C}))</th>
<th>Lead time ((L))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LTL</td>
<td>15</td>
<td>50</td>
<td>0</td>
<td>15</td>
<td>20</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Non refrigerated FTL</td>
<td>13</td>
<td>50</td>
<td>U[45,55]</td>
<td>25</td>
<td>50</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Refrigerated FTL</td>
<td>10</td>
<td>50</td>
<td>U[45,55]</td>
<td>25</td>
<td>50</td>
<td>2.5</td>
<td>2</td>
</tr>
</tbody>
</table>

Figures 2-4 are obtained by solving the problem using the weighted sum method. In these experiments we use \(\lambda_1 = 1\) and \(\lambda_2 = 0.7\). The sum of \(\lambda_1\) and \(\lambda_2\) is greater than 1. As shown in Figure 6, we change the value of \(\lambda_2\) and consequently the relative value of the multipliers during the experiments. In order to evaluate the impact of product shelf life on replenishment decisions we run the model for different values of \(k = 1, 2, \ldots, 10\). Since demands and fixed costs are random, for each problem we generate 10 problem instances, and present here the average of the results over all instances.

Figure 2: Replenishment mode selection at different levels of demands (same rate of variability).
Figure 2 presents the relationship between supplier selection decisions and product lifetime. This relationship is evaluated for different levels of demand. We make the following observations. First, as demand increases, the volume shipped using less-than-truckload shipments from a local supplier decreases and, concurrently, the amount shipped using full-truckload shipments increases. This shift makes sense since a higher demand level justifies the use of full truckloads. Using full truckloads from wholesale suppliers results in savings due to discounted replenishment costs. Second, as the length of the shelf life of a product increases, the use of full-truckload shipments increases. As the value of $k$ increases, the facility has the flexibility to use local suppliers as needed, or use discounted full-truckload shipments.

Figure 3 presents the relationship between product shelf life and costs. We investigate inventory holding costs, purchasing costs, order setup costs, and cargo container costs for different values of $k$. The results in Figure 3(a) indicate that, as the product shelf life increases, purchasing costs decrease. Long product shelf life provides more flexibility for inventory replenishment decisions. For example, as shelf life increases, the plant has the option to order not only from local suppliers, but also from suppliers located further away. As a result, purchasing costs decrease due to increasing the pool of suppliers. Figure 3(b) indicates that order costs decrease when $k$ increases. This is due to the fact that, as product shelf life increases, orders are placed less frequently. Each order is of a larger size, which justifies the use of full truckload shipments. Thus, cargo container costs increase with $k$ (Figure 3(c)). Figure 3(d) indicates that inventory holding costs increase due to the increase in product shelf life and, consequently, due to the increase in order size. The graphs in Figures 3(a) to 3(d) indicate similar trends for different levels of demand.
Based on the graphs in Figures 3(a) to 3(d), purchasing costs, order cost and inventory holding costs are highest when demand is low. When demand is low, the least expensive supplier – supplier 3 – is used for \( k > 5 \). For medium and high demand levels, this supplier is used when \( k > 2 \). Consequently, replenishment costs are higher when the demand level is low. Low levels of demand do not always justify the use of full-truckloads; thus, cargo costs are smaller. However, as the value of \( k \) increases, it makes sense to use supplier 3 and order full-truckloads which result in higher inventory when demand is low.

Figure 4 presents the total unit cost versus product shelf life for low, medium and high demand. The graphs indicate that the total unit cost decreases as product shelf life increases. Considering the relationships observed in Figure 3, it is clear that the decrease in replenishment and order costs – as product shelf life increases – is larger than the corresponding increase in cargo and inventory holding costs. As a result, as the product shelf life increases, total cost per unit decreases. Based on the results shown in Figure 4, as the value of \( k \) increases from 1 to 10, the total unit cost decreases 37%, 18%, and 23%, for low, medium, and high demand levels, respectively. This indicates that replenishment costs for perishable items are typically higher than for non-perishable products. The graph in Figure 4 also indicates that the total unit cost is highest when demand is low. This is mainly because low levels of demand do not justify using full-truckload shipments, which are less expensive than less-than-truckload shipments.

Figure 5 presents the relationship between the total emissions and product shelf life for low, medium and high demand levels. The graph indicates an increase in emissions as the product shelf life increases. As the value of \( k \) increases, transportation related emissions increase because of the increase in the volume shipped from supplier 3, who is located further away (see Figure 2). Figure 3(d) indicates an increase in the inventory holding costs with \( k \) and therefore, an increase in the inventory level. Therefore, emissions due to storage increase due to this increase in the inventory level.

![Figure 4: Total unit cost versus product shelf life for different levels of demand.](image)

Figure 6 presents the relationship between the total emissions and the value of \( \lambda_2 \) when the demand level is high. This is the multiplier we use for emissions in the objective function when implementing the weighted sum method. The higher the value of this multiplier, the higher will be the weight that CO\(_2\) emissions have in the objective function. For products with very short shelf life (\( k = 1 \)), increasing the value of \( \lambda_2 \) does not impact total emissions. When product shelf life is short, the facility has less flexibility in replenishment decisions. In our example, the facility replenishes its inventories using the local supplier. As the value of \( k \) increases, the facility has more options to explore. In this case, the multiplier \( \lambda_2 \) plays a greater role in reducing emissions in the supply chain. Increasing the value of \( \lambda_2 \) impacts replenishment decisions and reduces emissions. One can think of \( \lambda_2 \) as a penalty multiplier for emissions in the objective function, or as an emissions tax, which is charged per unit of CO\(_2\) generated in the supply chain. Thus, as the value of this tax increases, the amount of CO\(_2\) emitted decreases.
6. Summary and Conclusions
This paper presents a bi-objective mixed integer programming model to manage inventory replenishment decisions for fixed-shelf life perishable products during a fixed planning horizon. These are products such as dairy, canned products, or pharmaceuticals which have an expiration date. The demand of each period is known. The model minimizes costs and CO$_2$ emissions due to inventory replenishment in a facility.

The paper presents a numerical example and provides a number of results from the experimentations. The results indicate that as the shelf life of the product increases, replenishment and order costs decrease; and inventory and cargo cost increase. Total costs decrease as the shelf life of a product increase. These results indicate that replenishment costs for perishable products are higher than for non-perishable products. This result makes sense since a facility has more flexibility in selecting suppliers when managing non-perishable products - which have no fixed shelf life.

Experimental results also indicate that replenishment costs for slow moving perishable products are higher than replenishment costs for fast moving perishable products. Slow moving products have low demand, consequently, when managing these products a facility cannot take advantage of economies of scale offered for purchasing in larger quantities. Replenishment costs and the challenges in managing these inventories increase as the shelf life of a product decreases.

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